CSU FRESNO MATHEMATICS FIELD DAY
APRIL 18, 2009
MAD HATTER MARATHON 11-12
PART I

1. The remainder when $f(x) = 4x^4 + 10x^3 + kx^2 + bx - 2$ is divided by $x + 1$ is -3, when $f(x)$ is divided by $x - 1$ the remainder is 25. What is $k + b$?

   (a) 4
   (b) 5
   (c) 9
   (d) 13
   (e) None of the above.

2. How many distinct solutions does the equation

$$\sqrt{\sqrt{x}} = \sqrt[3]{x}$$

   have?

   (a) 1
   (b) 2
   (c) 3
   (d) 4
   (e) None of the above.

3. If the length of the hour and minute hand of a circular clock are 3 *in* and 4 *in* respectively, what is the distance in inches, between the tips of the hands at three o'clock?

   (a) 25 inches
   (b) 5 inches
   (c) 2.5 inches
   (d) It cannot be determined
   (e) None of the above.

4. Apple has found out that when x millions of dollars are spent on research, the profit, in millions of dollars, is

$$p(x) = 9 + 5\log_3(x + 1).$$

   How much would you say should be spent on research to make a profit of 24 million dollars?

(a) 2 million dollars

(b) 10 million dollars

(c) 27 million dollars

(d) $\sqrt{3}$ million dollars

(e) None of the above.

5. N is a three-digit odd number that is a multiple of five. All the digits are different and add up to 11. When you subtract the first digit from the second you get a negative number. What is N if you know all digits are different from zero?

(a) 245

(b) 425

(c) 605

(d) 515

(e) None of the above.

6. A sum of money is divided among three sons. If the first son gets 30% of the whole sum, the second son gets 40% of the remainder, and the third son gets the remaining $\$21,000$, what is the total sum of money?

(a) $\$70,000$

(b) $\$35,000$

(c) $\$50,000$

(d) $\$7,000$

(e) None of the above.

7. A square, a circle, and an equilateral triangle all have the same perimeter. Which shape has the largest area?

(a) Triangle

(b) Square

(c) Circle

(d) Both the square and the circle

(e) None of the above.

8. The product

$$\tan 1^\circ \tan 2^\circ \tan 3^\circ \cdots \tan 88^\circ \tan 89^\circ$$

is equal to

(a) $-\sqrt{3}$

(b) $\sqrt{3}$

(c) 1

(d) -1

(e) None of the above.

9. What type of solutions does the equation $x^3 + x^2 + x + 1 = 0$ have?

   (a) One real, two complex.

   (b) One complex, two real.

   (c) All real

   (d) All complex

10. Let $3^a = 4, 4^b = 5, 5^c = 6, 6^d = 7, 7^e = 8, 8^f = 9$. What is the value of the product $abcdef$?

    (a) 2

    (b) $\dfrac{1}{2}$

    (c) $\log 2$

    (d) $\log\left(\dfrac{1}{2}\right)$

    (e) None of the above.

11. Line ℓ_1 has equation $y = mx + p$. Line ℓ_1 crosses the y-axis at P and line ℓ_2 crosses the x-axis at $Q(q, 0)$. If the segment $\overline{PQ}$ is perpendicular to both lines, then what is the y-intercept of ℓ_2?

    (a) $-mq$

    (b) $-p$

    (c) $p - q$

    (d) $\sqrt{p^2 + q^2}$

    (e) None of the above.

12. Five members of a basketball team are weighed and an average weight is recalculated after each member is weighed. If the average increases by one pound each time, how much heavier is the last player than the first one?

    (a) 4 pounds

    (b) 8 pounds

    (c) 16 pounds

    (d) 32 pounds

    (e) None of the above.

13. A hollow cylinder of iron with height 32cm and internal and external radii 4cm and 5cm respectively is melted to form a solid sphere. Find the radius of the sphere.

    (a) 6
    (b) 288 π
    (c) 12
    (d) 144 π
    (e) None of the above.

14. Find the last two digits of 2009^{2009}.

    (a) 99
    (b) 01
    (c) 00
    (d) 89
    (e) None of the above.

15. The owner of a jewelry store hired 3 guards to protect his diamonds, but a thief still got in and stole some diamonds. On the way out, the thief met each guard, one at a time. To each he gave 1/2 of the diamonds he had then, and 2 more besides. He escaped with one diamond. How many did he steal originally?

    (a) 8 diamonds
    (b) 16 diamonds
    (c) 32 diamonds
    (d) 36 diamonds
    (e) None of the above.

16. Find the area of triangle ABC where $\angle ABC = 35°$, $\angle CAB = 35°$, length $BC = 5\ in$, and length of AB is 6.

    (a) 6 in^2
    (b) 12 in^2
    (c) 15 in^2
    (d) 24 in^2
    (e) None of the above.

17. Which of the numbers 2^{100}, 3^{50}, $10,000^2$, 500^3 is the largest?

    (a) 2^{100}
    (b) 3^{50}

(c) $10,000^2$

(d) 500^3

18. There is a trough that is 7 feet long, and its vertical cross sections are inverted isosceles triangles with a height 5 feet and base 2 feet.

    What is the volume of the trough?

    (a) 5

    (b) 10

    (c) 35

    (d) 70

    (e) None of the above.

19. $2 + 4 + 6 + 8 + \cdots + 2010 = ?$

    (a) $2,022,060$

    (b) $2,020,050$

    (c) $1,011,030$

    (d) $1,010,025$

    (e) None of the above.

20. A ladder leans against a vertical wall. The top of the ladder is 5 yards above the ground. When the bottom of the ladder is moved 2 yards farther away from the wall, the top of the ladder rests against the foot of the wall. What is the length of the ladder?

    (a) 7.25 yards

    (b) 5.25 yards

    (c) 7 yards

    (d) 8.5 yards

    (e) None of the above.

21. A fox chasing a hare, which has a start of 45 feet, jumps 3 feet every time the hare jumps 2 feet. In how many leaps does the fox overtake the hare if the fox jumps first?

    (a) 15

    (b) 16

    (c) 43

    (d) 45

    (e) None of the above.

22. Two fair dice are thrown. What is the probability that the sum of the numbers shown is 8?

    (a) $\frac{3}{36}$
    (b) $\frac{5}{36}$
    (c) $\frac{1}{6}$
    (d) $\frac{8}{36}$
    (e) None of the above.

23. A convex polygon has 200 sides. How many different *diagonals* (lines connecting non-adjacent vertices) can be drawn in this figure?

    (a) 200
    (b) 100
    (c) $20,000$
    (d) $19,700$
    (e) None of the above.

24. Start with an equilateral triangle of side 1. Form a new triangle by joining the midpoints of the sides. Then form a third triangle by joining the midpoints of the sides of the second triangle. Continue in this fashion *ad infinitum*. The sum of the perimeters of all the triangles is:

    (a) 6
    (b) 3^2
    (c) 3^3
    (d) ∞
    (e) None of the above.

25. Five real numbers are chosen and put in order from smallest to largest. The average of all five is 14 . The average of the three middle numbers is only 13 . What is the average of the largest and smallest ?

    (a) 15.5
    (b) 15
    (c) 14.5
    (d) 14
    (e) None of the above.

26. In an English-speaking village of 2029 inhabitants, at least x of the residents have the same two-letter initials. The least possible value of x is

    (a) 6
    (b) 5
    (c) 4
    (d) 3
    (e) None of the above.

27. If $\sin x + \cos x = 1.2$, then $\sin 2x =$

    (a) 0.6
    (b) 0.22
    (c) 2.4
    (d) 0.44
    (e) None of the above.

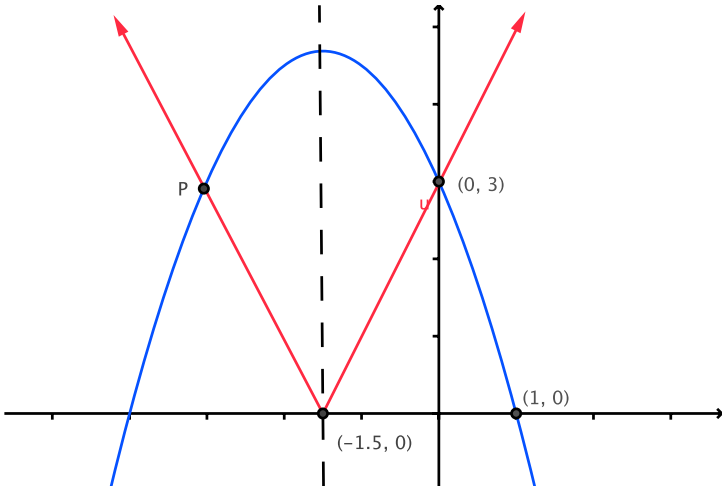
28. An old and somewhat illegible invoice shows that 72 canned hams were purchased for a total of $\$x67.9y$ (before tax), where x and y are digits. What is x?

    (a) 2
    (b) 3
    (c) 4
    (d) 6
    (e) None of the above.

29. Use the picture below to solve this problem. If the dashed line is the axis of the blue parabola and the red V-shaped graph corresponds to

$$f(x) = \left| x + \frac{3}{2} \right|$$

What are the coordinates of P?



(a) $(-3, 3)$

(b) $(3, 3)$

(c) $(-4.5, 3)$

(d) It cannot be determined.

(e) None of the above.

30. Find the maximum diameter of a circle that lies completely on or above the x-axis, passes through the origin, and intersects the parabola $y = x^2$ only at the origin.

(a) 1

(b) 2

(c) 0

(d) All diameters are possible.

(e) None of the above.

31. A game is played by one person with two fair coins as follows. The player makes four throws in succession. Each throw consists of tossing the two coins simultaneously. If at least one of the throws results in both coins showing "heads", the player wins; otherwise the player loses. What is the probability that the player wins?

(a) $\left(\frac{1}{4}\right)^4$

(b) $1 - \left(\frac{1}{4}\right)^4$

(c) $\left(\frac{3}{4}\right)^4$

(d) $1 - \left(\frac{3}{4}\right)^4$

(e) None of the above.

32. If a rectangular box has sides, front and bottom faces with areas of $2x, y/2$ and xy in^2 respectively, what is the volume of the solid in cubic inches?

(a) $2x^2y$

(b) x^2y^2

(c) $\frac{xy^2}{2}$

(d) xy

(e) None of the above.

33. Four friends, X, Y, Z and W, bought a motorbike for \$60.00. X paid one half of the sum of the amounts paid by the other boys, Y paid one third of the sum of the amounts paid by the other boys; and Z paid one fourth of the sum of the amounts paid by the other boys. How much did W have to pay?

    (a) 12
    (b) 13
    (c) 15
    (d) 20
    (e) None of the above.

34. Today is Saturday April 18^{th}, 2009. What day of the week will April 18^{th}, 2017 be?

    (a) Sunday
    (b) Monday
    (c) Tuesday
    (d) Wednesday
    (e) Saturday

35. A Chevy starts traveling east along a road. At the same time, from the same point a Ford starts travelling north at a speed 60 mi/hr. After one hour and twenty minutes, the cars are 100 mi apart. At what speed is the Chevy traveling?

    (a) It cannot be determined.
    (b) 75 mi/hr
    (c) 60 mi/hr
    (d) 45 mi/hr
    (e) None of the above.

36. If 2^{2009} is computed, then what is the units digit in the resulting integer?

    (a) 2
    (b) 4
    (c) 6
    (d) 8
    (e) 0

37. If $a + b = 2$ and $a^2 + b^2 = 5$, then what is the value of $a^3 + b^3$?

    (a) 7
    (b) 10

(c) 11

(d) 8

(e) None of the above.

38. Seven people are sitting in a theater watching a show. The row they are in contains seven seats. After intermission, they return to the same row but choose seats randomly. What is the probability that neither of the people sitting in the two aisle seats was previously sitting in an aisle seat?

(a) $\frac{5!}{7!}$

(b) $\frac{5!2!}{7!}$

(c) $\frac{4}{21}$

(d) $\frac{10}{21}$

(e) None of the above.

39. A man decided to go gambling one night. He has k dollars. He pays \$5.00 admission to enter a casino. While there he doubles his money. He leaves and pays \$5.00 for parking. He pays \$5.00 to enter a second casino. He again doubles his money, but when he pays \$5.00 for parking, he has no money left. Find k.

(a) \$11.25

(b) \$25

(c) \$45

(d) \$20.5

(e) None of the above.

40. The product of three consecutive integers is $35904 = 2^6 \cdot 3 \cdot 11 \cdot 17$. What is the sum of the integers ?

(a) 99

(b) 33

(c) 66

(d) 111

(e) None of the above.

ANSWER KEY

1	D
2	B
3	B
4	E
5	B
6	C
7	C
8	C
9	A
10	A
11	A
12	B
13	A
14	D
15	D
16	B
17	A
18	C
19	C
20	A
21	C
22	B
23	D
24	A
25	A
26	C
27	D
28	B
29	A
30	A
31	D
32	D
33	B
34	C
35	D
36	A
37	C
38	D
39	A
40	A

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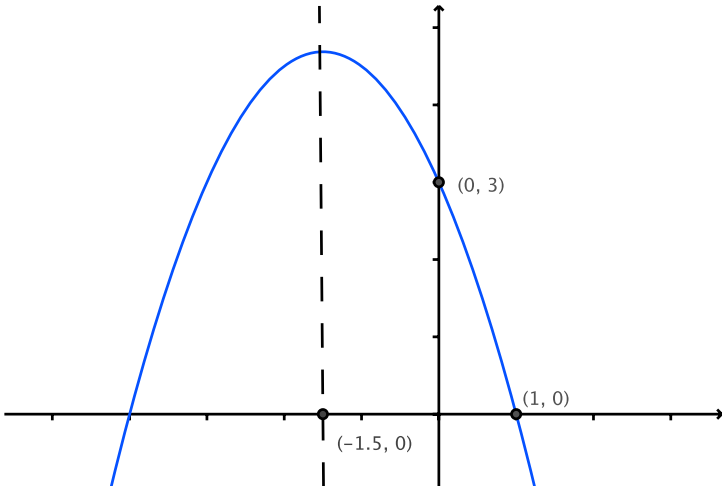
PART II

1. Use the picture below to solve this problem. If the dashed line is the axis of the parabola. What is the equation of the parabola?



   (a) $4y = -3x^2 - 9x + 12$
   (b) $y = -3x^2 - 9x + 12$
   (c) $4y = -x^2 - 9x + 3$
   (d) $y = -x^2 - 9x + 3$
   (e) None of the above.

2. For what values of x will $y_1 \geq y_2$, where

$$y_1 = 4x - 2 \qquad \textit{and} \qquad y_2 = \frac{1}{4}x - 1$$

   (a) $x \geq \frac{15}{4}$
   (b) $x \leq \frac{15}{4}$
   (c) $x \geq \frac{4}{15}$
   (d) $x \leq \frac{4}{15}$
   (e) None of the above.

3. The angles of a quadrilateral have degree measures that are four consecutive odd numbers. What is the degree measure of the smallest angle?

   (a) 85

(b) 87

(c) 88

(d) 89

(e) None of the above.

4. Let α be an acute angle such that $\cos(\alpha) = \frac{4}{5}$. Find $\tan(2\alpha)$.

(a) $\frac{7}{2}$

(b) $\frac{12}{5}$

(c) $\frac{24}{7}$

(d) $\frac{3}{2}$

(e) None of the above.

5. A cup contains 5 blue balls, 8 red balls and 2 white balls. 5 balls are chosen at random from the cup without replacement. What is the probability that at least one red ball is chosen ?

(a) $1 - \frac{1+2\binom{5}{4}+\binom{5}{3}}{\binom{15}{5}}$

(b) $\frac{1+2\binom{5}{4}+\binom{5}{3}}{\binom{15}{5}}$

(c) $1 - \frac{2\binom{5}{4}\binom{5}{3}}{\binom{15}{5}}$

(d) $\frac{2\binom{5}{4}\binom{5}{3}}{\binom{15}{5}}$

(e) None of the above.

6. I am thinking of a two-digit number less than 50. If you double my number and subtract 12, you get the original number with the digits reversed! What is the sum of the digits of the original number ?

(a) 21

(b) 84

(c) 48

(d) 12

(e) None of the above.

7. The value of $\sin^2(10^\circ) + \sin^2(20^\circ) + \sin^2(30^\circ) + \cdots + \sin^2(90^\circ)$ is

(a) $\frac{5}{2}$

(b) 5

(c) $\frac{\sqrt{2}}{2}$

(d) $5\frac{\sqrt{2}}{2}$

(e) None of the above.

8. Every graduating senior from Alpha High enters Fresno State, as do one half of those from Beta High. At Fresno State, one half of the freshman class are from Alpha High. If 400 students graduate from Beta High and 300 from Alpha, how many of Fresno State's freshmen are neither from Alpha nor Beta High?

   (a) 500

   (b) 400

   (c) 100

   (d) 50

   (e) None of the above.

9. You and your best friend Alex are playing a game that involves flipping a coin. Alex has just flipped ten heads in a row. You, being a bright person, are quite suspicious, but Alex assures you that the coin is a fair coin. If Alex isn't lying, what is the probability that the next toss of the coin will be heads?

   (a) $\frac{1}{2}$

   (b) $\left(\frac{1}{2}\right)^{10}$

   (c) $\left(\frac{1}{2}\right)^{11}$

   (d) $1 - \left(\frac{1}{2}\right)^{11}$

   (e) None of the above.

10. The graphs with equations

$$y = mx \qquad \textit{and} \qquad y = \frac{5}{2x+3}$$

intersect at $x = 1$ and $x = a$. Find the value of a.

   (a) $a = -\frac{5}{2}$

   (b) $a = \frac{5}{2}$

(c) $a = -\frac{2}{5}$

(d) $a = \frac{2}{5}$

(e) None of the above.

11. Find all the values of x for which: $\log_6(x-3)(x-7) = 1$.

(a) $x = 5 \pm \sqrt{10}$

(b) $x = 3$ and $x = 7$

(c) $x = 3$ and $x = -1$

(d) $x = 6^3$ and $x = 6^7$

(e) None of the above.

12. How many different factors does the number 10! have?

(a) 2^6

(b) 2^{15}

(c) 10

(d) 270

(e) None of the above.

13. 3 dice are thrown (one red, one white, and one black). In how many ways can we obtain a sum of 15? (note that because of the different colors, a toss of $6, 6, 3$ is different from a toss of $6, 3, 6$).

(a) 3

(b) 5

(c) 10

(d) 3!

(e) None of the above.

14. Robert has worn the same hat for d years. If he wears it for 12 more years, he would have worn it for d^2 years. What is the value of d?

(a) 3

(b) 4

(c) 7

(d) 12

(e) None of the above.

15. There is a group of boys and girls standing in line at the bus stop. Exactly 15 girls get on the first bus to arrive (and nobody else); now there are 2 boys for each girl still at the bus stop. On the second bus, exactly 45 boys get on (and nobody else); now, at the bus stop, there are 5 girls for each boy. What was the original number of girls?

(a) 15

(b) 25

(c) 40

(d) 50

(e) None of the above.

16. Solve the equation $(x^2 - 2x)^2 + 2(x^2 - 2x) + 1 = 0$

(a) $x = -1$

(b) $x = 0$

(c) $x = 1$

(d) $x = \frac{1}{2}$

(e) None of the above.

17. Solve for x

$$2^{x-1} \cdot \sqrt{2^x} = \frac{1}{32}$$

(a) $x = 12$

(b) $x = -\frac{8}{3}$

(c) $x = -4$

(d) $x = 4$

(e) None of the above.

18. How many subsets of $\{1, 2, 3, \cdots, 9, 10\}$ have a non-empty intersection with $\{1, 2, 3, 4\}$?

(a) 6!

(b) 2^6

(c) 960

(d) $10! - 6!$

(e) None of the above.

19. A surveyor places a base line along one bank of a river. From each end of the base line, a rock is sighted on the opposite bank of the river right on front of the base line. The base line is 60 yards long and the lines of sight of the rock form angles of 60^o with the base line. How wide is the river?

(a) 60

(b) 30

(c) $60\sqrt{3}$

(d) $30\sqrt{3}$

(e) None of the above.

20. How many natural numbers less than 108 are there, with sum of digits equal to 7 ?

(a) 5

(b) 8

(c) 9

(d) 10

(e) None of the above.

21. A parabola with vertex $(2, 0)$ and axis of symmetry parallel to the y-axis, passes through $(3, 1)$ and $(-3, t)$. Find the value of t.

(a) 1

(b) 4

(c) 16

(d) 25

(e) None of the above.

22. What is the largest power of 2 that divides $2^{2009} + 18^{2009}$?

(a) 2^{2100}

(b) 2^{2009}

(c) 2^{2010}

(d) 2^{3000}

(e) None of the above.

23. Solve for x and y (real numbers):

$$x(2 + i) + y(1 + i) = 3 + 2i$$

(a) $x = -1$ and $y = -1$

(b) $x = -1$ and $y = 1$

(c) $x = 1$ and $y = -1$

(d) $x = 1$ and $y = 1$

(e) None of the above.

24. In Fresno many fields have circular irrigation systems. A single sprinkler is placed on the center of the field. If the sprinkler sprays water just to touch the four edges of the field, what percentage of the field is not watered?

(a) 0%

(b) $\frac{4-\pi}{4}\ \%$

(c) $(100 - 25\pi)\%$

(d) It cannot be determined with the data given.

(e) None of the above.

25. In a local pet store, the birds sold for \$10.00 each and the rabbits sold for \$15.00 each. The total value was \$360.00. One night, the owner forgot to close the door and the animals got loose. Two birds and half of the rabbits disappeared. If the total value of the remaining pets was now \$220.00, how many of each were present before the escape?

(a) 16 rabbits and 12 birds.

(b) 12 rabbits and 16 birds.

(c) 10 rabbits and 15 birds.

(d) 15 rabbits and 10 birds.

(e) None of the above.

26. What is the measure of the acute angle between the hour and minute hands of a correctly 'real life' working clock at $4:18$?

(a) 21

(b) 2

(c) 4

(d) 18

(e) None of the above.

27. A right triangle in the first quadrant is bounded by lines $y = 0, y = x$, and $y = -x + 5$. Find its area.

(a) 25

(b) 10

(c) $\frac{25}{4}$

(d) $\frac{25}{2}$

(e) None of the above.

28. The Dodgers are playing The A's in the World series. Each team has an even chance to win any given game. The World Series is won by the first team to win 4 games, so at most seven games can be played. The Dodgers are leading the series 3 to 2. What is the probability that The Dodgers win the series?

(a) 0.75

(b) 0.5

(c) 0.333333...

(d) 0.25

(e) None of the above.

29. Suppose that $f(x) = ax + b$, where a and b are real numbers. Given that $f(f(f(x))) = 8x + 21$, what is the value of $a + b$?

(a) 2

(b) 3

(c) 5

(d) 8

(e) None of the above.

30. If $f(x)$ is a function that satisfies $f(2x + 1) = 2f(x) + 1$ for all x, and if $f(0) = 2$, then $f(3) =$

(a) 2

(b) 6

(c) 5

(d) 11

(e) None of the above.

31. If the product of the ages of a group of teenagers is $10584000 = 2^6 \cdot 3^3 \cdot 5^3 \cdot 7^2$, then the sum of their ages is

(a) 74

(b) 89

(c) 210

(d) 108

(e) None of the above.

32. Suppose $P(x)$ is a polynomial such that $P(1) = 1$ and

$$\frac{P(2x)}{P(x+1)} = 8 - \frac{56}{x+7}$$

for all real x for which both sides are defined. Find $P(0)$.

(a) 0

(b) 1

(c) 56

(d) $\frac{1}{8}$

(e) None of the above.

33. Suppose that 1 and 2 are roots of $x^3 + ax^2 + bx + c = 0$ and that $a + b = -15$. Then, $a =$

(a) 0

(b) 4

(c) 14

(d) -26

(e) None of the above.

34. Two circles of radius 2 are drawn so that each circle passes through the center of the other. What is the perimeter of the region of overlap?

(a) $\frac{4\pi}{3}$

(b) 4π

(c) 2π

(d) $\frac{8\pi}{3}$

(e) None of the above.

35. If x and y are distinct numbers that satisfy $2009 + x = y^2$ and $2009 + y = x^2$, then what is the value of xy?

(a) 2008

(b) -2008

(c) 2009

(d) -2009

(e) None of the above.

36. How many 5-digit numbers with all digits non-zero and no digit repeated are divisible by 25?

(a) 624

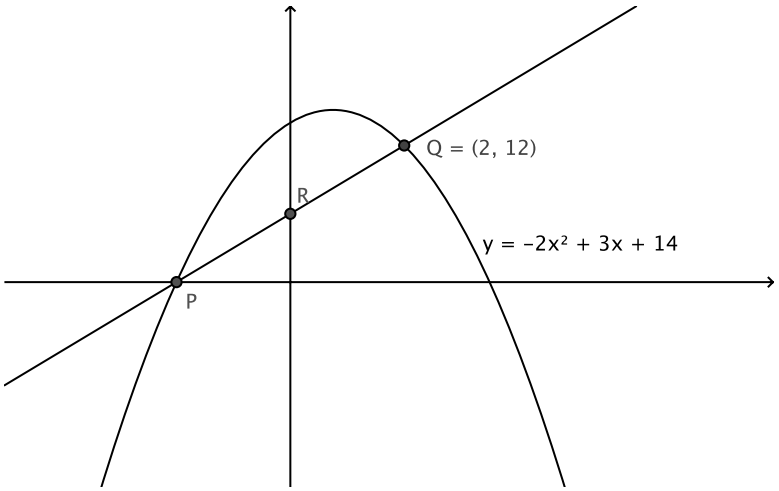
(b) 420

(c) 210

(d) 70

(e) None of the above.

37. Use the picture below to solve this problem. If P belongs to the parabola, the x-axis and the 'diagonal' straight line through Q, then what are the coordinates of R?



(a) $(-2, 0)$
(b) $(0, 6)$
(c) $(0, 2)$
(d) It cannot be determined.
(e) None of the above.

38. What is the last digit d of the 9-digit number $19700019d$, given that the number is prime?

(a) 1
(b) 3
(c) 5
(d) 7
(e) 9

39. Solve for x

$$2^x - 2^{x-2} = 3$$

(a) $x = -2$
(b) $x = 0$
(c) $x = 1$
(d) $x = 2$
(e) None of the above.

40. Suppose that $f(x + 1) = f(x) + 5$, and $f(0) = 2$. Find $f(500)$

(a) 502
(b) 1002
(c) 2502
(d) 3502
(e) None of the above.

ANSWER KEY

1	A
2	C
3	B
4	C
5	A
6	D
7	B
8	C
9	A
10	A
11	A
12	D
13	C
14	B
15	C
16	C
17	B
18	C
19	D
20	C
21	D
22	C
23	D
24	C
25	A
26	A
27	C
28	A
29	C
30	D
31	B
32	A
33	B
34	D
35	B
36	B
37	B
38	A
39	D
40	C