

CSU FRESNO MATHEMATICS FIELD DAY  
APRIL 18, 2009  
MAD HATTER MARATHON 9-10  
PART I

1. What is

$$2 + 3 \times 4 - 8 \div 2?$$

- (a) 3
  - (b) 6
  - (c) 10
  - (d) 16
2. A product costing \$20 is reduced in price by 25%. Later, a clearance sale offers an additional 40% reduction in the price. What is the total percentage reduction in price of the item?
- (a) 35%
  - (b) 55%
  - (c) 65%
  - (d) 90%
3. The owner of a castle has decided to build a moat around his castle. If the moat is to be circular in shape with inner radius 2 feet and outer radius 4 feet, and if it is to be 10 feet deep, what is the volume of water he needs to fill his moat?
- (a)  $12\pi$  cubic feet
  - (b)  $40\pi$  cubic feet
  - (c)  $120\pi$  cubic feet
  - (d)  $160\pi$  cubic feet

4. Simplify

$$\frac{2^3 8^{-\frac{2}{3}}}{4}$$

- (a)  $\frac{1}{2}$
  - (b) 1
  - (c) 2
  - (d) 8
5. If  $\triangle ABC$  is similar to  $\triangle DEF$ , determine the length DE if  $m\angle BAC = m\angle DEF$ ,  $BC = 2$ ,  $DF = 4$ , and  $AC = 1$ .
- (a) 1
  - (b) 2
  - (c) 4

- (d) It cannot be determined from the information given.
6. Find the fifth term of  $(x - y)^{10}$ .
- $210x^6y^4$
  - $-210x^6y^4$
  - $210x^5y^5$
  - $-210x^5y^5$
7. If  $a + b = 5$ ,  $a^2 - b^2 = 15$ , and  $a^2 + b^2 = 35$ , find  $a$  and  $b$ .
- $a = 1, b = 4$
  - $a = 4, b = 1$
  - $a = 5, b = 0$
  - none of the above
8. What is the sum of all multiples of 3 between 1 and 100?
- 297
  - 561
  - 1683
  - 15150
9. Place the following in order from smallest to largest:  $\log_2 32$ ,  $\log_3 81$ ,  $e^{\ln 3^2}$ , and  $e^{\ln 2^3}$ .
- $e^{\ln 3^2}, e^{\ln 2^3}, \log_2 32, \log_3 81$
  - $\log_3 81, \log_2 32, e^{\ln 2^3}, e^{\ln 3^2}$
  - $\log_2 32, \log_3 81, e^{\ln 2^3}, e^{\ln 3^2}$
  - $e^{\ln 2^3}, e^{\ln 3^2}, \log_2 32, \log_3 81$
10. Given the quadratic polynomial  $3x^2 + 9x - 2$ , which of the following statements describes the roots?
- The roots are complex conjugates.
  - There is one positive repeated real root.
  - There are two distinct positive real roots.
  - There is one positive real root and one negative real root.
11. Determine the point(s) of intersection, if any, of the line  $y - x = 4$  with the curve defined by  $x^2 + y^2 = 4$ .
- The line does not intersect the curve.
  - The line intersects the curve at two points:  $(-2, 0)$  and  $(2, 0)$ .
  - The line intersects the curve at one point:  $(2, 0)$ .
  - The line intersects the curve at two points:  $(0, -2)$  and  $(0, 2)$ .

12. Find the sum  $\binom{3}{3} + \binom{4}{3} + \binom{5}{3}$ .

(a)  $\binom{6}{3}$

(b)  $\binom{6}{4}$

(c)  $\binom{12}{3}$

(d)  $\binom{12}{9}$

13. Which of the following is a true statement about prime numbers?

(a) The sum of two prime numbers is always a prime number.

(b) The difference of two prime numbers is always a prime number.

(c) The sum of two prime numbers is never a prime number.

(d) None of the above

14. Determine the fraction denoted by  $0.23\overline{18}$ .

(a)  $\frac{2295}{9900}$

(b)  $\frac{2295}{9990}$

(c)  $\frac{2295}{9999}$

(d)  $\frac{2318}{10000}$

15. What is the radius of a circle that circumscribes a right triangle with legs of length 6 and 8?

(a) 8

(b) 6

(c) 5

(d) 10

16. Determine the  $y$ -intercept of perpendicular bisector of the line segment with endpoints  $(-3, 4)$  and  $(-9, 16)$ .

(a) -2

(b) 7

(c) 13

(d) 26

17. Simplify  $\sqrt[3]{-27x^{-5}y^{12}}$
- (a)  $-3x^{-15}y^{36}$
  - (b)  $-3x^{-\frac{5}{3}}y^4$
  - (c)  $-3x^{\frac{5}{3}}y^{-4}$
  - (d)  $3x^{-\frac{5}{3}}y^4$
18. What is the radius of the circle inscribed in a right triangle with sides of length 6 cm, 8 cm, and 10 cm?
- (a) 2 cm
  - (b) 2.5 cm
  - (c) 3 cm
  - (d) 4 cm
19. Let  $a_n = \frac{1}{n} - \frac{1}{n+1}$ . Find the sum  $a_1 + a_2 + \dots + a_{1000}$ .
- (a)  $\frac{999}{1000}$
  - (b)  $\frac{1000}{1001}$
  - (c) 1
  - (d)  $\frac{1}{2}$
20. What is the largest possible area of a rectangular plot of land that you can enclose with 120 ft. of wire?
- (a) 720 sq. ft.
  - (b) 800 sq. ft.
  - (c) 900 sq. ft.
  - (d) 3,600 sq. ft.
21. Working together, Paul and Dave can paint their fence in 3 hours and 45 minutes. Last year, Paul painted the fence alone and he took 4 hours more than Dave did when he painted the fence himself the previous year. How long did Paul take to paint the fence alone?
- (a) 4 hours
  - (b) 6 hours
  - (c) 8 hours
  - (d) 10 hours
22. Fence posts are being placed at 12-foot intervals along a road that is 1,440 feet long. How many fence posts are needed?
- (a) 119
  - (b) 120
  - (c) 121
  - (d) 122

23. Find the greatest common divisor of 72 and 343.

- (a) 1
- (b) 3
- (c) 6
- (d) 12

24. Solve the inequality

$$2x - \frac{8}{3x} < \frac{2}{3}.$$

- (a)  $-1 < x < \frac{4}{3}$
- (b)  $x < -1$
- (c)  $x > \frac{4}{3}$
- (d)  $x < -1$  and  $0 < x < \frac{4}{3}$

25. What is  $\sqrt{40}$  to the nearest hundredth?

- (a) 6.25
- (b) 6.32
- (c) 6.43
- (d) 6.58

26. Coming home from a meeting, Rick gets a ride from a friend part of the way and rides his bike the remainder of the way. Rick's friend drives an average of 60 miles per hour and Rick bikes an average of 10 miles per hour. If it takes Rick 5 hours to travel the 200 miles home, how far did he ride his bike?

- (a) 20 miles
- (b) 30 miles
- (c) 3 miles
- (d) 2 miles

27. If  $x$  is directly proportional to  $y$  and inversely proportional to the square of  $z$ , find  $x$  when  $y = 24$  and  $z = 6$  if  $x = 6$  when  $y = 8$  and  $z = 2$ .

- (a) 2
- (b)  $\frac{2}{9}$
- (c) 9
- (d) 18

28. How many different total scores can be obtained from two rolls of two dice?

- (a) 11
- (b) 21

- (c)  $11^2$
- (d)  $6^4$

29. A sphere with radius  $r$  has volume  $A$ . If the volume of the sphere is doubled, what is the new radius in terms of  $r$ .

- (a)  $2r$
- (b)  $8r$
- (c)  $\sqrt[3]{2}r$
- (d)  $\frac{r}{\sqrt[3]{2}}$

30. How many 4-digit numbers greater than 4,000 can be formed from the digits 2, 3, 4, 5, and 6 if no digit is repeated?

- (a) 72
- (b) 24
- (c) 12
- (d) 3

31. Of the 25 city council members, 12 voted for Proposition A, 19 voted for Proposition B, and 10 voted for both Propositions. Which of the following statements must be true?

- I Seven members voted for Proposition B but not for Proposition A.
- II Four members voted against both Proposition A and Proposition B.
- III Eleven members voted for one Proposition and against the other.

- (a) I only
- (b) II only
- (c) I and II only
- (d) II and III only

32. In the correctly worked out addition problem below, what is the greatest possible value for digit B?

$$\begin{array}{r} A6 \\ B4 \\ +B8 \\ \hline 148 \end{array}$$

- (a) 9
- (b) 8
- (c) 6
- (d) 5

33. In the sequence below, each term after the first term,  $x$ , is found by doubling the preceding term and then subtracting 3. Determine the value of  $x + z$ .

$$x, y, 19, 35, 67, z, \dots$$

- (a) 133.5  
(b) 138  
(c) 139  
(d) 142
34. Define  $\boxed{a} = 2\sqrt{a+1}$  for all nonnegative integers  $a$ . What is  $\boxed{a} \times \boxed{a}$ ?
- (a) 2  
(b) 4  
(c)  $4\sqrt{a+1}$   
(d)  $4(a+1)$
35. If  $2a + b = 7$  and  $b + 2c = 23$ , determine the average of  $a, b$ , and  $c$ .
- (a) 5  
(b) 7.5  
(c) 10  
(d) 12.25
36. If  $n$  is an even integer, what is the remainder when  $(n+1)^2$  is divided by 4?
- (a) 3  
(b) 2  
(c) 1  
(d) 0
37. A cube with side length 10 rests inside a sphere so that each of the eight vertices of the cube touch the surface of the sphere. What is the volume of this sphere?
- (a)  $\frac{500}{3}\pi$   
(b)  $500\sqrt{3}\pi$   
(c)  $\frac{4000}{3}\pi$   
(d)  $4000\sqrt{3}\pi$
38. Every 14 days Bill mows the lawn and every 8 days Alice weeds the garden. On May 1, Bill was mowing the lawn and Alice was weeding the garden. What is the next date on which they will both be doing these chores?
- (a) May 15  
(b) May 29

- (c) June 12
  - (d) June 26
39. If the sum of the squares of two numbers is equal to the product of their sum and their difference, then the product of the two numbers must be:
- (a) 0
  - (b) 1
  - (c) 4
  - (d) 6
40. For how many different integer values of  $b$  are both roots of  $x^2 + bx - 16 = 0$  integers?
- (a) 3
  - (b) 4
  - (c) 5
  - (d) 6

MAD HATTER MARATHON 9-10  
PART I ANSWERS

1. c
2. b
3. c
4. a
5. b
6. a
7. d
8. c
9. b
10. d
11. a
12. b
13. d
14. a
15. c
16. c
17. b
18. a
19. b
20. c
21. d
22. c
23. a
24. d
25. b
26. a

27. a

28. b

29. c

30. a

31. d

32. c

33. b

34. d

35. a

36. c

37. b

38. d

39. a

40. c

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PART II

1. What is the last digit of the number  $(2008^{2008})^{2008}$ ?

- (a) 2
- (b) 4
- (c) 6
- (d) 8

2. What is the product of the solutions of the equation

$$4(2^{2x}) - 17(2^x) + 4 = 0?$$

- (a) 1
- (b)  $-\frac{5}{2}$
- (c) -1
- (d) -4

3. Two telephone poles  $x$  and  $y$  feet tall are placed  $M$  feet apart. Lines are drawn from the top of each pole to the bottom of the other pole. How many feet along the ground from the pole that is  $x$  feet high will the lines from the two poles intersect?

- (a)  $\frac{Mx}{x-y}$
- (b)  $\frac{Mx}{x+y}$
- (c)  $\frac{x}{Mx+My}$
- (d)  $\frac{My}{x+y}$

4. The lengths of the sides of a triangle are the roots of  $x^3 - 12x^2 + 47x - 60$  and they are all natural numbers. What is the area of the triangle?

- (a) 5
- (b) 6
- (c) 7
- (d) 8

5. If  $p, q,$  and  $r$  are positive integers and  $p + \frac{1}{q + \frac{1}{r}} = \frac{41}{19}$ , then  $r$  equals

- (a) 1
- (b) 2
- (c) 3
- (d) 4

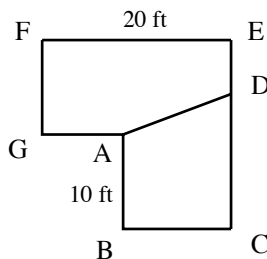
6. Today is Saturday. What day of the week will it be 100 days from now?
- (a) Monday
  - (b) Tuesday
  - (c) Wednesday
  - (d) Sunday
7. The time on a digital clock is 5:55. How many minutes will pass before the clock next shows a time with all digits identical?
- (a) 316
  - (b) 255
  - (c) 72
  - (d) 71
8. In an election for class president, 125 votes are cast by students who are voting to choose one of four candidates. Each student must vote for only one candidate, and the candidate with the highest number of votes is the winner. What is the least number of votes the winner can receive?
- (a) 31
  - (b) 32
  - (c) 51
  - (d) 62
9. A chocolate drink is 8% pure chocolate. If 10 ounces of milk are added to 30 ounces of the drink, what is the percent of chocolate in the new drink?
- (a) 6
  - (b) 8
  - (c) 10
  - (d) 15
10. The surface area of a large cube is 5400 square inches. This cube is cut into a number of identical smaller cubes, each having a volume of 216 cubic inches. How many smaller cubes are there?
- (a) 180
  - (b) 164
  - (c) 125
  - (d) 64
11. The wholesale price of a CD is \$10.00. If the store initially increases the price by 40% (to make a profit) and then reduces the price by 30%, what is the final price?
- (a) \$19.80

- (b) \$17.00
- (c) \$9.80
- (d) \$9.00

12. In how many ways can 75 be expressed as the sum of two or more consecutive integers?

- (a) 2
- (b) 3
- (c) 4
- (d) 5

13. In the diagram,  $ABCDEFG$  is a room having corners at right angles, with  $EF = 20$  ft.,  $AB = 10$  ft., and  $AG = GF$ . The total area of the room is 280 square feet. A wall is built from  $A$  to  $D$  creating two rooms of equal area. What is the distance in feet between  $C$  and  $D$ ?



- (a) 15
- (b)  $\frac{50}{3}$
- (c) 12
- (d)  $\frac{40}{3}$

14. For the points  $(0, 2)$ ,  $(6, 6)$  and  $(10, 0)$  which of the following statements are true?

- I The points form the vertices of a right triangle.
- II The points form the vertices of an isosceles triangle.
- III The largest angle is at the vertex located at  $(6, 6)$ .

- (a) All are true
- (b) I and II
- (c) II and III
- (d) Only III

15. Three different numbers are chosen so that when each of the numbers is added to the average of the remaining two, the result is 65, 69, and 76. The average of the original three numbers is

- (a) 34

- (b) 35  
(c) 36  
(d) 37
16. The statement  $|x + 1| + 2|x - 2| \leq 6$  is equivalent to
- (a)  $-1 \leq x \leq 3$   
(b)  $-1 \leq x \leq 2$   
(c)  $x \leq 3$   
(d)  $x \leq -1$  or  $x \geq 2$
17. What is the maximum number of regions into which a circle can be divided by four different chords?
- (a) 7  
(b) 10  
(c) 11  
(d) 12
18. On the math portion of the SAT, Alma scored in the 75th percentile, Betty in the 80th percentile, and Cristina in the 90th percentile on the test. What percentage of the students taking the math SAT scored better than Alma and Betty but worse than Cristina?
- (a) 5%  
(b) 10%  
(c) 15%  
(d) 75%
19. A set  $S$  contains ten distinct integers between 1 and 25, inclusive. Half of the numbers are even. Half of the numbers are divisible by 3. Half of the numbers are divisible by 5. How many of the numbers are divisible by 10?
- (a) 4  
(b) 3  
(c) 2  
(d) 1
20. Suppose that  $3^a = 4$ ,  $4^b = 5$ ,  $5^c = 6$ ,  $6^d = 7$ ,  $7^e = 8$ , and  $8^f = 9$ . Determine the product  $abcdef$ .
- (a) 3  
(b)  $\sqrt{6}$   
(c) 2  
(d) 1

21. How many solutions  $(x, y, z, w)$  are there to the following system of equations?

$$\begin{aligned}x + y + z &= w \\ \frac{1}{x} + \frac{1}{y} + \frac{1}{z} &= \frac{1}{w}\end{aligned}$$

- (a) 0
  - (b) 1
  - (c) 2
  - (d) infinitely many
22. The average age of 5 people in a room is 30. One of the people who is aged 40 leaves the room. What is the average age of the remaining 4 people?
- (a) 30
  - (b) 27.5
  - (c) 25
  - (d) 20
23. An  $n$ -story apartment building has a single penthouse on the  $n$ th floor. Each floor has twice as many apartments as the floor directly above it. How many apartments are on the first floor?
- (a)  $2^{n-1}$
  - (b)  $2^n$
  - (c)  $2^{n+1}$
  - (d)  $n^2$
24. The squares of two consecutive positive integers differ by 2008. What is the sum of these two positive integers?
- (a) 2006
  - (b) 2007
  - (c) 2008
  - (d) It cannot be determined from the given information.
25. The probability that it will be sunny tomorrow is  $\frac{1}{4}$ . If it is sunny, the probability that Susanna will play tennis is  $\frac{4}{5}$ . If it is not sunny, the probability that Susanna will play tennis is  $\frac{2}{5}$ . What is the probability that Susanna will not play tennis tomorrow?
- (a)  $\frac{2}{5}$
  - (b)  $\frac{1}{10}$
  - (c)  $\frac{9}{20}$
  - (d)  $\frac{1}{2}$

26. The people living on Main Street have all decided to buy new house numbers, so they lined up at the hardware store in order of their address (i.e., 1, 2, 3, ...). If the store has 100 of each digit, what is the first address that won't be able to buy its house number?
- (a) House #100
  - (b) House #111
  - (c) House #113
  - (d) House #163
27. Points  $A$  and  $B$  are 5 units apart. How many lines that lie in a given plane containing  $A$  and  $B$  are 2 units from  $A$  and 3 units from  $B$ ?
- (a) 1
  - (b) 2
  - (c) 3
  - (d) more than 3
28. Suppose you have a large bar of chocolate, scored into 200 individual squares to make up a rectangular slab 10 pieces wide and 20 pieces long. What is the least number of breaks you need to make to divide the slab into 200 pieces?
- (a) 200
  - (b) 199
  - (c) 100
  - (d) 50
29. A rubber ball is dropped from a height of 3 feet, and it bounces back up and continues to bounce up and down. Each time the ball bounces up, it returns to a height of  $\frac{2}{3}$  of that from which it fell. What is the total distance travelled by the ball in feet?
- (a) 18
  - (b) 15
  - (c) 12
  - (d) 9
30. 5 cows can eat 2 acres of grass in 10 days. How many days will it take 10 cows to eat 6 acres of grass?
- (a) 30 days
  - (b) 25 days
  - (c) 20 days
  - (d) 15 days

31. Simplify  $(2 + i)^2(2 - i)$ .

- (a)  $10 + 5i$
- (b)  $6 + 3i$
- (c)  $2 + 5i$
- (d)  $10 - 5i$

32. Find the product

$$\frac{8}{4} \cdot \frac{12}{8} \cdot \frac{16}{12} \cdots \frac{4n + 4}{4n} \cdots \frac{2008}{2004}$$

- (a) 252
- (b) 502
- (c) 1004
- (d) 2008

33. Certain positive integers have the following properties:

- I The sum of the squares of their digits is 50; and
- II Each digit is larger than the one to its left.

The product of the digits of the largest integer having these properties is

- (a) 25
- (b) 36
- (c) 48
- (d) 60

34. A sphere is inscribed inside a cube that has a surface area of 24 square meters. A second cube is then inscribed within the sphere. What is the surfact area in square meters of the inner cube?

- (a) 3
- (b) 6
- (c) 8
- (d) 9

35. Tom's age is  $T$  years, which is also the sum of the ages of his three children. His age  $N$  years ago was twice the sum of their ages then. What is the value of  $\frac{T}{N}$ ?

- (a) 5
- (b) 4
- (c) 3
- (d) 2

36. A set of 25 blocks is arranged into a  $5 \times 5$  square. How many different combinations of 3 blocks can be selected from that set so that no two are in the same row or column?

- (a) 2300
- (b) 600
- (c) 125
- (d) 100

37. Assuming that  $x < 0$ , what is

$$\sqrt{\frac{x}{1 - \frac{x-1}{x}}}$$

- (a)  $-x$
- (b)  $x$
- (c) 1
- (d)  $x\sqrt{-1}$

38. What is the tens digit in the sum  $7! + 8! + \dots + 2008!$ ?

- (a) 6
- (b) 4
- (c) 3
- (d) 1

39. At a party, there were 200 people, of whom 5% wore one earring and of the other 95%, half wore two earrings and half wore no earrings. How many earrings were worn at the party?

- (a) 50
- (b) 100
- (c) 150
- (d) 200

40. For how many real numbers  $y$  does the equation

$$(x^2 - 1)^2 = y$$

have an odd number of real-valued solutions for  $x$ ?

- (a) 0
- (b) 1
- (c) 2
- (d) infinitely many

MAD HATTER MARATHON 9-10  
PART II ANSWERS

1. c
2. d
3. b
4. b
5. c
6. a
7. a
8. b
9. a
10. c
11. c
12. d
13. d
14. a
15. b
16. a
17. c
18. b
19. c
20. c
21. d
22. b
23. a
24. c
25. d
26. d

27. c

28. b

29. b

30. d

31. a

32. b

33. d

34. c

35. a

36. b

37. a

38. b

39. d

40. b