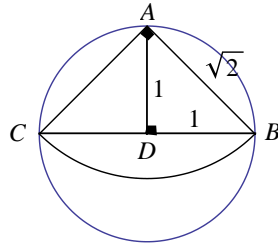


Solution. (b) We may assume the radius of the circle is equal to 1 unit., so its area is equal to π square units. Since $m\angle A = 90^\circ$, \overline{BC} is a diameter of the circle. The radius of the circle sector is then $\sqrt{2}$, and so its area is $\frac{1}{4}\pi(\sqrt{2})^2 = \pi/2$ square units. The ratio is then $\pi/(\pi/2) = 2$.



13. Successive discounts of 10%, 20% and 30% is equivalent to a single discount of _____.
- (a) 49.9% (b) 49.8%
- (c) 49.7% (d) 49.6%
- (e) None of these

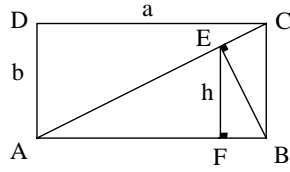
Solution. (d) It is easiest if you look at the resulting discount on 100 dollars. The first discount (10%) results in a price of $100 - (.1)100 = 90$ (dollars). The second discount results in a price of $90 - (.2)90 = 72$ (dollars). The third discount results in a price of $72 - (.3)72 = 50.4$ (dollars). So the savings on 100 dollars is 49.6 dollars, which is a discount of 49.6%.

14. In exactly how many minutes after 12 noon do the minute and hour hands of a clock form a 180° angle for the first time?
- (a) $32\frac{8}{11}$ (b) $32\frac{9}{11}$
- (c) $32\frac{10}{11}$ (d) 33
- (e) None of these

Solution. (a) The minute hand will travel $6m$ degrees in m minutes. The hour hand moves 12 times as slow, and so will travel through $6m/12 = m/2$ degrees in m minutes. The difference between the two is $6m - m/2 = (11/2)m$. Solve the equation $(11/2)m = 180$ to get $m = 360/11 = 32\frac{8}{11}$ minutes.

Alternate Solution. The hour hand and minute hand will point in the same direction exactly 11 times during a 12 hour period in equally spaced intervals. So

17. In the figure below, $ABCD$ is a rectangle, $a = DC = AB$ and $b = AD = BC$. Also, $\overline{EB} \perp \overline{AC}$ and $\overline{EF} \perp \overline{AB}$. Determine $h = EF$ as a function of a and b .



- (a) $\frac{a^2b}{\sqrt{a^2 + b^2}}$ (b) $\frac{a^2b}{a^2 + b^2}$
 (c) $\frac{ab^2}{a^2 + b^2}$ (d) $\frac{ab^2}{\sqrt{a^2 + b^2}}$
 (e) None of these

Solution. (b) First we note that $\triangle EFB \sim \triangle BEC$, so

$$\frac{h}{EB} = \frac{EB}{b} \implies h = \frac{(EB)^2}{b} \tag{1}$$

Secondly, we have $\triangle BEC \sim \triangle ABC$, to obtain

$$\frac{EB}{b} = \frac{a}{AC} \implies EB = \frac{ab}{AC} \tag{2}$$

Combine equations 1 and 2 to get

$$\begin{aligned} h &= \frac{(EB)^2}{b} \\ &= \frac{(ab/AC)^2}{b} \\ &= \frac{a^2b}{(AC)^2} \\ &= \frac{a^2b}{a^2 + b^2}, \end{aligned}$$

where the last equality was obtained by the Pythagorean Theorem applied to $\triangle ABC$.

18. Suppose Alex and Betty can mop the floor in 3 hours, Betty and Charles can mop the floor in 2 hours and Charles and Diana can mop the floor in 4 hours. How long would it take Alex and Diana to mop the floor?
- (a) 6 hours (b) 8 hours
 (c) 10 hours (d) 12 hours
 (e) None of these

Solution. (d) Let a, b, c and d be the fraction of floor mopped by Alex, Betty, Charles and Diana respectively in one hour. Then we obtain the equations

$$\begin{aligned} a + b &= \frac{1}{3} \\ b + c &= \frac{1}{2} \\ c + d &= \frac{1}{4}. \end{aligned}$$

Now subtract the second equation from the sum of the first and third to obtain

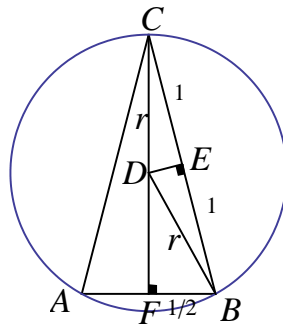
$$a + d = \frac{1}{12}.$$

So it will take Alex and Diana 12 hours to mop the floor.

19. Determine the radius of the circumscribing circle about the isosceles triangle whose respective side lengths are 2, 2 and 1.

- (a) $\frac{4}{\sqrt{15}}$ (b) $\frac{5}{\sqrt{15}}$
(c) 1 (d) $\frac{\sqrt{5}}{2}$
(e) None of these

Solution. (a) The figure below represents the triangle, $\triangle ABC$, and circumscribing circle.



\overline{ED} is the perpendicular bisector of \overline{AC} and \overline{CF} is the perpendicular bisector of \overline{AB} . Thus, D is the center of the circle, and consequently $DC = DB = r$ is the radius. We first note that $\triangle CDE \sim \triangle CBF$, and so

$$\frac{r}{DE} = \frac{2}{1/2} = 4.$$

Additionally, $DE = \sqrt{r^2 - 1}$ by the Pythagorean Theorem. So

$$\begin{aligned}\frac{r}{\sqrt{r^2 - 1}} = 4 &\implies r = 4\sqrt{r^2 - 1} \\ &\implies r^2 = 16(r^2 - 1) \\ &\implies r^2 = \frac{16}{15} \\ &\implies r = \frac{4}{\sqrt{15}}.\end{aligned}$$

20. How many solutions, $0 \leq x \leq 2\pi$, are there to the equation

$$\sin^2 x + \cos^2 x + \tan^2 x + \cot^2 x + \sec^2 x + \csc^2 x = 8?$$

(a) 2 (b) 4

(c) 6 (d) 8

(e) None of these

Solution. (d) If we let $s^2 = \sin^2 x$, then using the standard identities and definitions, the equation becomes

$$1 + \frac{s^2}{1 - s^2} + \frac{1 - s^2}{s^2} + \frac{1}{1 - s^2} + \frac{1}{s^2} = 8.$$

Rearrange this equation to obtain

$$(3s^2 - 2)(3s^2 - 1) = 0.$$

This leads us to

$$\sin x = \pm\sqrt{\frac{2}{3}} \quad \text{and} \quad \sin x = \pm\sqrt{\frac{1}{3}}$$

A quick glance at the plot of $y = \sin x$ for $0 \leq x \leq 2\pi$ instantly shows that each of these equations has 4 solutions. So there are 8 solutions in all.