

CSU FRESNO MATHEMATICS FIELD DAY  
APRIL 22, 2006  
MAD HATTER MARATHON 11-12  
PART I

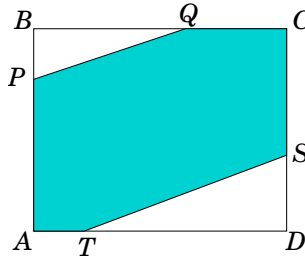
1. Find  $1 + 2 + 3 + \dots + 2005 + 2006$ .
  - (a) 4,017
  - (b) 2,012,018
  - (c) 2,013,021
  - (d) 4,024,036
  - (e) None of the above
  
2. Today is Saturday. What day of the week will it be exactly 2006 days from today?
  - (a) Monday
  - (b) Tuesday
  - (c) Wednesday
  - (d) Thursday
  - (e) Friday
  
3. Solve for  $x$ :  $3^{5x+2} = \sqrt{27}$ .
  - (a)  $-\frac{4}{15}$
  - (b)  $-\frac{1}{10}$
  - (c)  $\frac{2}{5}$
  - (d)  $\frac{4}{5}$
  - (e) None of the above
  
4. How many integers between 1000 and 2000 have all three of the numbers 15, 20, and 25 as factors?
  - (a) 1

- (b) 2
- (c) 3
- (d) 4
- (e) 5

5. Find the coefficient of  $x^2y^3$  in the expansion of  $(3x - 2y)^5$ .

- (a)  $-720$
- (b)  $-72$
- (c) 180
- (d) 1440
- (e) None of the above

6. Figure  $ABCD$  is a rectangle with  $AB = 8$  and  $BC = 10$ . Find the area of the shaded hexagon  $APQCST$  if  $AP = 6$ ,  $QC = 4$ ,  $CS = 5$ , and  $AT = 2$ .



- (a) 48
- (b) 55
- (c) 60
- (d) 62
- (e) None of the above

7. Find the product of the roots of  $x^3 - 2x^2 + 4x - 5 = 0$ .

- (a)  $-2$
- (b) 4
- (c) 5
- (d) 40

- (e) None of the above
8. From a regular deck of 52 cards, you draw a King and then a Queen. What is the probability that the next card you draw will also be a face card (i.e. Jack, Queen, or King)?
- (a) 0%
- (b) 5%
- (c) 8%
- (d) 12%
- (e) None of the above
9. Find  $\tan(300^\circ)$ .
- (a)  $-\sqrt{3}$
- (b)  $-\frac{1}{\sqrt{3}}$
- (c)  $\frac{\sqrt{3}}{2}$
- (d) 1
- (e) None of the above
10. If the lengths of one pair of opposite sides of a rectangle are increased by 20%, and the lengths of the second pair of opposite sides are decreased by 10%, by how much is the area of the rectangle changed?
- (a) The area is the same.
- (b) Increased by 10%
- (c) Increased by 15%
- (d) Decreased by 5%.
- (e) None of the above
11. Solve for  $z$ :  $(3 + 4i)z = 23 + 14i$ .
- (a)  $7\frac{2}{3} + 3\frac{1}{2}i$
- (b)  $\frac{13}{25} - 2i$

- (c)  $5 - 2i$   
(d)  $125 - 50i$   
(e) None of the above
12. Three vertices of parallelogram  $ABCD$  are  $A(-1, 1)$ ,  $B(4, 5)$ , and  $C(3, 1)$ . Find the coordinates of the fourth vertex  $D$ .
- (a)  $(-3, -4)$   
(b)  $(-2, -3)$   
(c)  $(1, 1)$   
(d)  $(7, 0)$   
(e) None of the above
13. Approximately what percentage of the first 10,000 natural numbers have a 1 somewhere in them?
- (a) 10%  
(b) 22%  
(c) 34%  
(d) 45%  
(e) None of the above
14. Find the area of an equilateral triangle inscribed in a circle of diameter 10.
- (a) 25  
(b)  $\frac{75}{2}$   
(c)  $25\sqrt{38}$   
(d)  $\frac{75\sqrt{3}}{4}$   
(e) None of the above
15. If  $\sin \alpha = \frac{1}{5}$  and  $\alpha$  lies in the second quadrant, in what quadrant is  $2\alpha$ ?
- (a) I  
(b) II

- (c) III  
(d) IV  
(e) Not enough information given.
16. Simplify:  $\log_2 \sqrt{\log_2 \sqrt{2\sqrt{2}}}$ .
- (a) 0  
(b) 1  
(c)  $\log_2 3$   
(d)  $\log_2 2\sqrt{2}$   
(e)  $\log_2 \sqrt{3} - 1$
17. A train leaves point A going at 40 miles per hour. An hour and a half later a car leaves from point A going in the same direction going at 70 miles per hour. How long will it take the car to catch up with the train?
- (a) 1 hour  
(b) 1 hour 15 minutes  
(c) 1 hour 45 minutes  
(d) 2 hours  
(e) None of the above
18. The ratio of the height of a cone to the diameter of its base is 3 : 4, and its volume is  $32\pi$ . Find the radius of the base.
- (a) 3  
(b) 4  
(c)  $2\pi$   
(d)  $\frac{3}{2}\pi$   
(e) None of the above
19. If  $\{a_n\}$  is an arithmetic sequence whose first term is 26 and 100-th term is 2006, find its 50-th term.
- (a) 1003

- (b) 1006
  - (c) 1016
  - (d) 1026
  - (e) None of the above
20. Solve for  $x$ :  $\frac{1}{x} + \frac{1}{2} = \frac{1}{x-1} - \frac{5}{6}$ .
- (a)  $-\frac{1}{2}$
  - (b)  $\frac{8}{9}$
  - (c)  $\frac{3}{2}, \frac{8}{9}$
  - (d)  $-\frac{1}{2}, \frac{3}{2}$
  - (e) None of the above
21. The product of two irrational numbers
- (a) is always a rational number
  - (b) is always an irrational number
  - (c) is either a rational number or an irrational number
  - (d) is either an irrational number or a complex number
  - (e) None of the above
22. The base of an isosceles triangle is 18 cm, and the lateral side is 15 cm. Find the radius of the inscribed circle.
- (a) 3 cm
  - (b) 3.5 cm
  - (c) 4 cm
  - (d) 4.5 cm
  - (e) 5 cm
23. If a polynomial with real coefficients has  $5 + 3i$  and  $-1$  as roots, then
- (a) 1 must also be a root.

- (b)  $5 - 3i$  must also be a root.
- (c)  $-5 - 3i$  must also be a root.
- (d) there need not be another root.
- (e) there must be another root but it is none of the above.
24. If you simultaneously flip three coins, what is the probability of seeing at least two heads?
- (a)  $\frac{1}{8}$
- (b)  $\frac{3}{8}$
- (c)  $\frac{1}{2}$
- (d)  $\frac{2}{3}$
- (e) None of the above
25. A chemist has a solution  $A$  which is 30% acid and another solution  $B$  which is 75% acid. How much of solution  $A$  should he mix with  $B$  to obtain 18 liters of a 50% solution?
- (a) 9 liters
- (b) 10 liters
- (c) 11 liters
- (d) 12 liters
- (e) None of the above
26. In the proportion  $x_1 : x_2 = x_3 : x_4$ , the sum of the first three numbers is 48. The second number is  $\frac{4}{3}$  of the first number, and the third number is  $\frac{1}{4}$  of the second number. Find the fourth number  $x_4$ .
- (a) 8
- (b) 12
- (c) 18
- (d) 24
- (e) None of the above

27. The sum of seven consecutive numbers is 126. Find the product of the smallest and the largest of these numbers.
- (a) 315
  - (b) 324
  - (c) 450
  - (d) 468
  - (e) None of the above
28. Each side of a rhombus is 13 cm, and one diagonal is 10 cm. Find the length of the other diagonal.
- (a)  $\sqrt{69}$  cm
  - (b) 12 cm
  - (c) 17 cm
  - (d) 24 cm
  - (e) None of the above
29. At a sandwich shop there are 5 kinds of bread, 4 kinds of cold cuts, 3 kinds of cheese, and 2 kinds of dressing. How many different sandwiches can be prepared using one kind of each of bread, cold cuts, cheese, and dressing?
- (a) 14
  - (b) 60
  - (c) 80
  - (d) 120
  - (e) None of the above
30. If  $a = 6$  and  $b = 24$ , find  $\frac{(a^8 - b^4)}{(a^4 + b^2)(a^2 + b)}$ .
- (a) 0
  - (b) 1
  - (c) 3
  - (d) 6
  - (e) 12

31. A bag contains 4 red, 3 white, and 5 blue marbles. You draw two marbles at random. What is the probability that both are red?
- (a)  $\frac{1}{11}$
  - (b)  $\frac{1}{9}$
  - (c)  $\frac{2}{9}$
  - (d)  $\frac{5}{18}$
  - (e) None of the above
32. The hypotenuse  $AB$  of a right triangle  $ABC$  is 6 and the height  $CH$  is  $\frac{4\sqrt{2}}{3}$ . Find the smaller leg of the triangle.
- (a) 2
  - (b) 3
  - (c)  $2\sqrt{2}$
  - (d)  $2\sqrt{3}$
  - (e) None of the above
33. Solve for  $x$ :  $|4x - 22| \leq 2006$
- (a)  $x \leq 507$
  - (b)  $x \leq -496$
  - (c)  $x \leq -496$  or  $x \geq 507$
  - (d)  $-496 \leq x \leq 507$
  - (e) None of the above
34. Which of the following statements is true?
- (a) The sum of two prime numbers is always prime.
  - (b) The sum of two composite numbers is always composite.
  - (c) The sum of two nonzero numbers is always nonzero.
  - (d) The sum of a positive number and a negative number is either positive or negative.

- (e) The sum of two rational numbers is always rational.
35. Kate and Helen have to make 60 pyramids for their geometry project. Kate can make them in 6 hours, and Helen can make them in 12 hours. Assuming that they will work at these rates, how long will it take them to make 60 pyramids together?
- (a) 3 hours  
(b) 4 hours  
(c) 4 hours 30 minutes  
(d) 5 hours  
(e) None of the above
36. If you roll four fair dice, what is the probability that all four numbers will be the same?
- (a)  $\frac{1}{1296}$   
(b)  $\frac{1}{216}$   
(c)  $\frac{1}{54}$   
(d)  $\frac{1}{36}$   
(e) None of the above
37. How much is 10% of 25% of 500?
- (a) 12.5  
(b) 17.5  
(c) 52.5  
(d) 200  
(e) None of the above
38. The angles of a quadrilateral have degree measures that are four consecutive odd numbers. What is the degree measure of the smallest angle?
- (a) 85  
(b) 87

- (c) 88
  - (d) 89
  - (e) None of the above
39. A boat travels 145 km downstream and it takes it the same time to travel 95 km upstream. The speed of the current is 5 km/h. Determine the speed of the boat in still water.
- (a) 22 km/h
  - (b) 23 km/h
  - (c) 24 km/h
  - (d) 25 km/h
  - (e) None of the above
40. The diagonals of a trapezoid divide it into four triangles. Find the area of the trapezoid if the areas of the two triangles adjacent to the bases of the trapezoid are  $S_1$  and  $S_2$ .
- (a)  $S_1 + S_2$
  - (b)  $S_1 \cdot S_2$
  - (c)  $S_1 + S_2 + S_1 \cdot S_2$
  - (d)  $\sqrt{S_1} \cdot \sqrt{S_2}$
  - (e)  $(\sqrt{S_1} + \sqrt{S_2})^2$

MAD HATTER MARATHON 11-12

PART I ANSWERS

- |       |       |
|-------|-------|
| 1. c  | 21. c |
| 2. c  | 22. d |
| 3. b  | 23. b |
| 4. c  | 24. c |
| 5. a  | 25. b |
| 6. d  | 26. a |
| 7. c  | 27. a |
| 8. e  | 28. d |
| 9. a  | 29. d |
| 10. e | 30. e |
| 11. c | 31. a |
| 12. b | 32. a |
| 13. c | 33. d |
| 14. d | 34. e |
| 15. d | 35. b |
| 16. e | 36. b |
| 17. d | 37. a |
| 18. b | 38. b |
| 19. b | 39. c |
| 20. d | 40. e |

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PART II

1. Solve for  $x$ :  $\frac{5x - 2}{x + 4} = \frac{3}{2}$ .

(a)  $\frac{6}{13}$

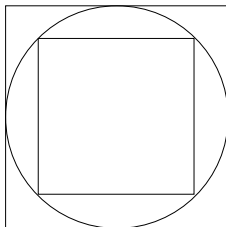
(b)  $\frac{6}{7}$

(c)  $\frac{16}{13}$

(d)  $\frac{16}{7}$

(e) None of the above

2. A square is inscribed in a circle which is inscribed in a bigger square. The area of the larger square is 64. Find the area of the smaller square.



(a)  $24\sqrt{2}$

(b)  $28\sqrt{2}$

(c) 32

(d) 36

(e) None of the above

3. Simplify:  $\frac{x^{-2}\sqrt{x^3}}{\sqrt[3]{x^5}}$ .

(a)  $x^{-\frac{13}{6}}$

(b)  $x^{-\frac{29}{15}}$

(c)  $x^{-\frac{11}{15}}$

(d)  $x^{\frac{7}{6}}$

(e) None of the above

4. If you deal three cards from a regular deck of 52 cards, what is the probability that at least one of them will be a face card (i.e. Jack, Queen, or King)?

(a)  $\approx 5\%$

(b)  $\approx 15\%$

(c)  $\approx 40\%$

(d)  $\approx 55\%$

(e) None of the above

5.  $\left(\frac{\sqrt{3}}{2} + \frac{1}{2}i\right)^{2006} =$

(a)  $\frac{1}{2} + \frac{\sqrt{3}}{2}i$

(b)  $\frac{1}{2} - \frac{\sqrt{3}}{2}i$

(c)  $-\frac{\sqrt{3}}{2} + \frac{1}{2}i$

(d)  $-\frac{\sqrt{3}}{2} - \frac{1}{2}i$

(e) None of the above

6. Find the area of an equilateral triangle whose perimeter is 10.

(a)  $5\sqrt{3}$

(b)  $\frac{100}{9}$

(c)  $\frac{25}{3\sqrt{3}}$

(d)  $\frac{50\sqrt{3}}{9}$

(e) None of the above

7. How many different factors does the number  $10!$  have?
- (a)  $2^6$
  - (b)  $2^{15}$
  - (c) 10
  - (d) 270
  - (e) None of the above
8. Find the value of  $A$  such that the equation  $x^2 + Ax + (A - 1) = 0$  has exactly one real root.
- (a)  $-1$
  - (b) 2
  - (c) 3
  - (d) 4
  - (e) None of the above
9. A collection of nickels, dimes, and quarters has a total value of \$2.55. There are twice as many nickels as dimes, and there are six more dimes than there are quarters. How many dimes are in the collection?
- (a) 7
  - (b) 8
  - (c) 9
  - (d) 10
  - (e) None of the above
10. In one condominium,  $\frac{2}{3}$  of the men are married to  $\frac{3}{5}$  of the women. What fraction of the condo residents is married?
- (a)  $\frac{4}{13}$
  - (b)  $\frac{12}{19}$
  - (c)  $\frac{19}{30}$

- (d)  $\frac{2}{5}$
- (e) None of the above
11. A computer password must consist of 8 symbols that can be either one of the 26 letters of the alphabet or one of the 10 digits 0 through 9. If the password is required to contain at least one letter, how many passwords are there?
- (a)  $36^8$
- (b)  $36^8 - 10^8$
- (c)  $26 \cdot 36^7$
- (d)  $8 \cdot 26 \cdot 36^7$
- (e) None of the above
12. The ratio of the volume of a sphere to its surface area is  $5 : 2$ . What is the radius of the sphere?
- (a)  $3\pi$
- (b) 5
- (c)  $\frac{10}{\sqrt{\pi}}$
- (d) 7.5
- (e) None of the above
13. Which of the numbers  $2^{100}$ ,  $3^{50}$ ,  $10,000^2$ ,  $500^3$  is the largest?
- (a)  $2^{100}$
- (b)  $3^{50}$
- (c)  $10,000^2$
- (d)  $500^3$
14. A bowler scores 146, 132, 153, 148, and 138 in five games. What must be bowled in the next game to attain an average of 150?
- (a) 190
- (b) 183
- (c) 160

- (d) 153
- (e) None of the above
15. Find an equation of the line that passes through the point  $(3, 4)$  and is parallel to  $6x - 2y = 7$ .
- (a)  $y = -3x - 13$
- (b)  $y = -3x - 5$
- (c)  $y = 3x - 9$
- (d)  $y = 3x - 5$
- (e) None of the above
16. There are 20 students in a class. The teacher wants to assign project  $A$  to 10 students and project  $B$  to the remaining 10 students. How many ways are there to assign these projects?
- (a) 20
- (b)  $20!$
- (c)  $\frac{20!}{10!}$
- (d)  $\frac{20!}{2 \cdot 10! \cdot 10!}$
- (e) None of the above
17. Simplify:  $8^{\log_4 12}$ .
- (a) 2
- (b) 512
- (c)  $24\sqrt{3}$
- (d)  $72\sqrt{2}$
- (e) None of the above
18. If  $a$ ,  $b$ ,  $a + b$ , and  $a - b$  are all prime numbers, which of the following statements must be true about the sum of these four numbers?
- (a) The sum is prime.
- (b) The sum is odd and divisible by 3.

- (c) The sum is odd and divisible by 7.
- (d) The sum is even but not divisible by 4.
- (e) The sum is divisible by 4.
19. Mike has six plastic sticks whose lengths are 1, 2, 3, 5, 10, and 15. How many different triangles can he make using these sticks for sides (one stick per side)?
- (a) 0
- (b) 2
- (c) 20
- (d) 216
- (e) None of the above
20. At a certain company, twice as many men as women apply for work. If  $\frac{1}{9}$  of the applicants is hired and  $\frac{1}{10}$  of the men is hired, what fraction of those who are hired is women?
- (a)  $\frac{2}{5}$
- (b)  $\frac{1}{4}$
- (c)  $\frac{1}{3}$
- (d)  $\frac{2}{15}$
- (e) None of the above
21. Next year Bob's dad will be three times as old as Bob. Nine years from now, his dad will be twice as old as Bob. How old is Bob now?
- (a) 7 years
- (b) 8 years
- (c) 10 years
- (d) 12 years
- (e) None of the above

22. The length of a rectangle is twice its width. If the length is increased by 5 and the width is decreased by 2, then the area remains the same. Find the area of the rectangle.
- (a) 50  
 (b) 200  
 (c) 240  
 (d) 300  
 (e) None of the above
23. In how many distinguishable ways can the letters of the word DODECAHEDRON be arranged?
- (a)  $12!$   
 (b)  $12^8$   
 (c)  $\frac{11!}{2!}$   
 (d)  $\frac{12!}{2 \cdot 2 \cdot 3}$   
 (e) None of the above
24. The circumference of the front tires of a car is 25 cm less than the circumference of the rear tires. If the front wheels make 800 revolutions in traveling 1 km, find the radius of the rear tires.
- (a)  $\frac{75}{\pi}$  cm  
 (b)  $\frac{90}{\pi}$  cm  
 (c)  $25\pi$  cm  
 (d)  $30\pi$  cm  
 (e) None of the above
25.  $\frac{123 \cdot 456 + 123 + 456}{123 + 456 \cdot 124} =$
- (a) 0.5  
 (b) 1

- (c) 2
- (d)  $\frac{123}{124}$
- (e) None of the above
26. How many real roots does the equation  $\sqrt{x^2 + 1} + \sqrt{x^2 + 2} = 4$  have?
- (a) 0
- (b) 1
- (c) 2
- (d) 3
- (e) 4
27.  $\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \dots + \frac{1}{2005 \cdot 2006} =$
- (a)  $\frac{2005}{2}$
- (b) 1003
- (c)  $\frac{2005}{2006}$
- (d)  $1 - \frac{1}{2005 \cdot 2006}$
- (e) None of the above
28. The diameter and the height of a cylinder are equal to the side of a cube. Find the ratio of the cylinder's volume to the cube's volume.
- (a)  $\frac{\pi}{4}$
- (b)  $\frac{3\pi}{8}$
- (c)  $\frac{2}{\pi}$
- (d)  $\frac{2\pi}{9}$
- (e) None of the above
29. Simplify:  $\frac{\sqrt{4} + \sqrt{6} + \sqrt{24}}{\sqrt{1} + \sqrt{6} + \sqrt{9} + \sqrt{150}}$ .

(a)  $\frac{1}{5}$

(b)  $\frac{1}{2}$

(c) 1

(d)  $\frac{4}{15}$

(e) None of the above

30. Find the smallest positive prime  $p$  such that  $p^3 + 10p^2$  is a perfect square. What is the sum of the digits of  $p$ ?

(a) 7

(b) 8

(c) 12

(d) 14

(e) None of the above

31. Find the fifth term of the geometric progression whose ratio is 3 and the sum of the first four terms is 80.

(a) 27.5

(b) 54

(c) 162

(d) 486

(e) None of the above

32. Solve for  $x$ :  $\ln 2 + \ln x = \frac{1}{3} \ln 3 + 4 \ln e$ .

(a)  $4e - 1$

(b)  $e^4 - 1$

(c)  $\frac{1 + 4e}{2}$

(d)  $\frac{\sqrt[3]{3}e^4}{2}$

(e) None of the above

33.  $ABC$  is a right triangle,  $C$  is its right angle,  $CH$  is the corresponding height, and  $CD$  is the corresponding bisector. The angle between  $CH$  and  $CD$  is  $26^\circ$ . Find the acute angles of the triangle  $ABC$ .
- (a)  $13^\circ, 77^\circ$
  - (b)  $19^\circ, 71^\circ$
  - (c)  $26^\circ, 64^\circ$
  - (d)  $38^\circ, 52^\circ$
  - (e) None of the above
34. An auto dealer paid \$2,000 for a used car which he is going to sell. He wants to price it so that he can offer a 20% discount and still make 15% of the price he paid for it. At what price should the car be marked?
- (a) \$2,875
  - (b) \$2,700
  - (c) \$2,645
  - (d) \$2,345
  - (e) None of the above
35. Find the largest possible number of diagonals that can be drawn in a polygon with 2006 sides.
- (a) 1,003
  - (b) 2,006
  - (c) 2,009,009
  - (d) 2,011,015
  - (e) 4,022,030
36. Which of the following statements is **not** true?
- (a) The product of two positive numbers is always a positive number.
  - (b) The product of two nonzero numbers is always a nonzero number.
  - (c) The product of two real numbers is always a real number.
  - (d) The product of two rational numbers is always a rational number.
  - (e) The product of two irrational numbers is always an irrational number.

37. The bases of an isosceles trapezoid are 22 cm and 18 cm, and the height is 10 cm. Find the radius of the circle circumscribed around the trapezoid.

- (a) 10 cm
- (b) 11 cm
- (c) 12 cm
- (d)  $2\sqrt{33}$  cm
- (e)  $\sqrt{130}$  cm

38. Evaluate:  $\frac{4351^2 - 4347^2}{4350 \cdot 4353 - 4351^2}$ .

- (a)  $\frac{1}{2}$
- (b) 1
- (c) 2
- (d) 4
- (e) 8

39. Simplify:  $\frac{\sin^4 x - \cos^4 x}{\cos x - \sin x}$

- (a)  $\sin^3 x - \cos^3 x$
- (b)  $\cos^3 x - \sin^3 x$
- (c)  $\sin x + \cos x$
- (d)  $-\cos x - \sin x$
- (e) None of the above

40. Solve for  $x$ :  $\log_2 x + \log_3 x = 3 + \log_2 3 + \log_3 4$ .

- (a)  $\frac{1}{6}$
- (b)  $\frac{2}{3}$
- (c)  $\frac{3}{2}$
- (d) 6
- (e) 12

MAD HATTER MARATHON 11-12

PART II ANSWERS

- |       |       |
|-------|-------|
| 1. d  | 21. a |
| 2. c  | 22. b |
| 3. a  | 23. c |
| 4. d  | 24. a |
| 5. a  | 25. b |
| 6. c  | 26. c |
| 7. d  | 27. c |
| 8. b  | 28. a |
| 9. c  | 29. b |
| 10. b | 30. b |
| 11. b | 31. c |
| 12. d | 32. d |
| 13. a | 33. b |
| 14. b | 34. a |
| 15. d | 35. c |
| 16. e | 36. e |
| 17. c | 37. e |
| 18. a | 38. e |
| 19. a | 39. d |
| 20. a | 40. e |