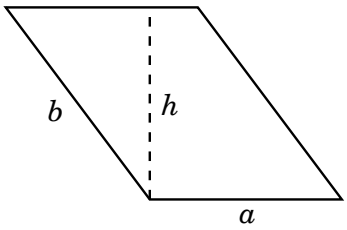
CSU FRESNO MATHEMATICS FIELD DAY
APRIL 19, 2008
MAD HATTER MARATHON 11-12
PART I

1. Determine the length of the diagonals of the parallelogram shown if $a = 6\ in,\ b = 10\ in$ and $h = 8\ in$.




**a)** $10\ in\ \ and\ \ 8\ in$
**b)** $10\ in\ \ and\ \ 6\ in$
**c)** $2\sqrt{137}\ in\ \ and\ \ 8\ in$
**d)** $4\sqrt{13}\ in\ \ and\ \ 8\ in$
**e)** $4\sqrt{13}\ in\ \ and\ \ 4\sqrt{13}\ in$

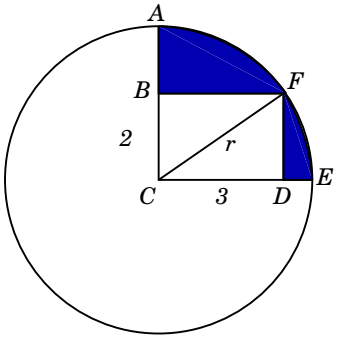
2. An urn contains three white and four black balls. We take out a ball and put it in a drawer without looking at it. After that we take out a second ball. Find the probability that this ball is white.

**a)** $\frac{1}{6}$
**b)** $\frac{3}{7}$
**c)** $\frac{5}{6}$
**d)** $\frac{1}{3}$
**e)** $\frac{1}{7}$

3. Determine the roots of $z^3 + 6z^2 - 4z - 24 = 0$

**a)** $-6,\ 4,\ -4$
**b)** $6,\ 4,\ -4$
**c)** $-6,\ 2,\ -2$
**d)** $6,\ 2,\ -2$

4. $C$ is the center of the circle and $F$ is a point on the circle such that $BCDF$ is a 2 in by 3 in rectangle. What is the area of the shaded region? (in square inches).




**a)** $\frac{13\pi}{2} - 5$
**b)** $\frac{13\pi}{4} - 5$
**c)** $\frac{13\pi}{2} - 6$
**d)** $\frac{13\pi}{4} - 6$

5. Evaluate

$$\left(b^{\frac{\log_{100} a}{\log_{10} a}} \cdot a^{\frac{\log_{100} b}{\log_{10} b}}\right)^{2\log_{ab}(a+b)}$$

**a)** $(a+b)^4$
**b)** $4(a+b)$
**c)** $\frac{1}{a+b}$
**d)** $\frac{1}{(a+b)^4}$
**e)** $a+b$

6. Find the $20^{th}$ term and the sum of the first 20 terms of the arithmetic progression $4, 9, 14, 19, \cdots$

**a)** $20^{th}\ term = 99 \quad sum = 1030$
**b)** $20^{th}\ term = 109 \quad sum = 1030$
**c)** $20^{th}\ term = 109 \quad sum = 1050$
**d)** $20^{th}\ term = 99 \quad sum = 1050$

7. Solve for $x$

$$9^x - 4 \cdot 3^{x+1} + 27 = 0$$

**a)** $x = 3 \quad and \quad x = 9$
**b)** $x = -1 \quad and \quad x = -2$
**c)** $x = 1 \quad and \quad x = 2$
**d)** $x = -3 \quad and \quad x = -9$

8. Find positive solutions to the system

$$\begin{aligned} \log_{10}(x^2+y^2) &= 2 \\ \log_2(x-2) &= \log_2 y \end{aligned}$$

**a)** $x = 8 \quad and \quad y = 6$
**b)** $x = 6 \quad and \quad y = 8$
**c)** $x = 2 + 2\sqrt{2^7+2} \quad and \quad y = 2\sqrt{2^7+2}$
**d)** $x = 2\sqrt{2^7+2} \quad and \quad y = 2 + 2\sqrt{2^7+2}$
**e)** *There are no positive solutions*

9. Solve for $x$

$$e^{2x} - 3e^x + 2 = 0$$

**a)** $x = -1 \quad x = -2$
**b)** $x = 1 \quad x = 2$
**c)** $x = e \quad x = e^2$
**d)** $x = 0 \quad x = \ln 2$
**e)** $x = 0$

10. If $f(x) = x^2$ and $g(x) = x - 3$, find the composite functions $f \circ g$ and $g \circ f$.

**a)** $(f \circ g)(x) = x^2 - 3^2 \quad and \quad (g \circ f)(x) = (x-3)^2$
**b)** $(f \circ g)(x) = x^2 - 3 \quad and \quad (g \circ f)(x) = (x-3)^2$
**c)** $(f \circ g)(x) = (x^2-3)^2 \quad and \quad (g \circ f)(x) = (x^2-3)^2$
**d)** $(f \circ g)(x) = (x-3)^2 \quad and \quad (g \circ f)(x) = x^2 - 3^2$
**e)** $(f \circ g)(x) = (x-3)^2 \quad and \quad (g \circ f)(x) = x^2 - 3$

11. Solve the equation in $[0, \pi)$

$$(1 + \sin 2x)^2 + (1 - \sin 2x)^2 = 5 - \sin 2x$$

**a)** $\frac{\pi}{2}$
**b)** $\frac{3\pi}{2}$
**c)** $\frac{3\pi}{4}$
**d)** $\frac{\pi}{4}$
**e)** *There are no solutions*

12. If $xy = 1$ and $x > 0$, which of the following statements is true?

**a)** When $x$ is greater than 1, $y$ is greater than 1
**b)** When $x$ is less than 1, $y$ is less than 1
**c)** When $x$ is greater than 1, $y$ is negative
**d)** As $x$ increases, $y$ increases
**e)** As $x$ increases, $y$ decreases

13. Assume that $\sin\theta = \frac{1}{\sqrt{3}}$ and $\theta$ is in the first quadrant. Evaluate $\tan\theta + 3\sin\theta$

**a)** $\sqrt{2} + \sqrt{3}$
**b)** $\sqrt{2} + \frac{1}{\sqrt{3}}$
**c)** $\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}}$
**d)** $\frac{1}{\sqrt{2}} + \sqrt{3}$

14. A sum of 35 integers, all of which are less than 100, is $S$. Two digits in one of the integers are interchanged and a new sum $T$ is produced. Then the difference $S - T$ is divisible by:

**a)** 2
**b)** 5
**c)** 7
**d)** 9
**e)** *None of the above*

15. A cube measuring 100 units on each side is painted only on the outside and cut into unit cubes. The number of cubes with paint only on two sides is

**a)** 1000
**b)** 1125
**c)** 1176
**d)** 980
**e)** *None of the above*

16. If the sum of the first and the fifth term of an arithmetic progression is equal to 10 and the product of the third term and the fifth term equals 50, find the $11^{th}$ term of the progression.

**a)** 15
**b)** 20
**c)** 25
**d)** 30

17. Solve the equation in $[0, \pi)$

$$\cot x + \sin x \cos x \left(1 + \tan^2 x\right) = 2$$

**a)** $\frac{\pi}{4}$
**b)** $\frac{\pi}{2}$
**c)** $\frac{3\pi}{4}$
**d)** $\frac{3\pi}{2}$
**e)** *There are no solutions*

18. If $f(x) = \sqrt{x}$ and $g(x) = \sqrt{2x}$, find the composite functions $f \circ g$ and $g \circ f$.

**a)** $(f \circ g)(x) = \sqrt[4]{2x}$ *and* $(g \circ f)(x) = \sqrt[4]{2x}$
**b)** $(f \circ g)(x) = \sqrt[4]{2x}$ *and* $(g \circ f)(x) = \sqrt{x}\sqrt[4]{2}$
**c)** $(f \circ g)(x) = \sqrt[4]{2x}$ *and* $(g \circ f)(x) = \sqrt{2}\sqrt[4]{x}$
**d)** $(f \circ g)(x) = \sqrt{x}\sqrt[4]{2}$ *and* $(g \circ f)(x) = \sqrt[4]{2x}$
**e)** $(f \circ g)(x) = \sqrt{2}\sqrt[4]{x}$ *and* $(g \circ f)(x) = \sqrt[4]{2x}$

19. Find the terms $a_5, a_6$ of the geometric progression

$$\sqrt{2}, a_2, a_3, 3\sqrt{6}, a_5, a_6, 27\sqrt{2}$$

**a)** $a_5 = \sqrt{6},\ a_6 = 3\sqrt{2}$
**b)** $a_5 = 3\sqrt{2},\ a_6 = \sqrt{6}$
**c)** $a_5 = 9\sqrt{6},\ a_6 = 9\sqrt{2}$
**d)** $a_5 = 9\sqrt{2},\ a_6 = 9\sqrt{6}$

20. For what values of $a$ does the system of equations

$$\begin{aligned} x^2 &= y^2 \\ (x-a)^2 + y^2 &= 1 \end{aligned}$$

have exactly 3 solutions?

**a)** *For all* $a \geq 0$
**b)** *For* $-2 \leq a \leq 2$
**c)** $a = \pm 1$
**d)** *For* $a \in \{-1, 0, 1\}$
**e)** *None of the above*

21. On a dark lonely night, a man is standing 5 yards away from a street light. The man is 2 yards tall and the light is 6 yards high. How long is the man's shadow?

**a)** $2 yards$
**b)** $4 yards$
**c)** $\frac{5}{2} yards$
**d)** $\frac{7}{3} yards$
**e)** *None of the above*

22. Find the $20^{th}$ term of the geometric progression $8, 4, 2, 1, \cdots$

**a)** $\frac{1}{2^8}$
**b)** $\frac{1}{2^{10}}$
**c)** $\frac{1}{2^{16}}$
**d)** $\frac{1}{2^{18}}$
**e)** $\frac{1}{2^{20}}$

23. If

$$y = \left| 2\sin\left(2\pi x - \frac{\pi}{2}\right) - 1 \right|$$

then the largest possible value for $y$ (for real $x$) is

**a)** 1
**b)** 2
**c)** 3
**d)** 0
**e)** *None of the above*

24. Solve the equation $2\sqrt{3x-1} = 3x$

**a)** $x = -\frac{2}{3}$
**b)** $x = \frac{2}{3}$
**c)** $x = 0$
**d)** $x = \frac{1}{3}$
**e)** $x = -\frac{1}{3}$

25. What is the value of the digit $d$ for which the number $d456d$ is divisible by 18?

**a)** 1
**b)** 2
**c)** 4
**d)** 8
**e)** 6

26. If $4^x - 4^{x-1} = 24$, then $(2x)^x$ is equal to

**a)** 25
**b)** 125
**c)** $25\sqrt{5}$
**d)** $5\sqrt{5}$
**e)** *None of the above*

27. Given $x > y > 0$, and $z \neq 0$, which of the following inequalities is not always correct?

**a)** $x - z > y - z$
**b)** $xz > yz$
**c)** $\frac{x}{z^2} > \frac{y}{z^2}$
**d)** $xz^2 > yz^2$
**e)** *None of the above*

28. Two parallel chords in a circle have lengths 6 and 8. The distance between them is 1. Then the diameter of the circle is

**a)** $10\sqrt{3}$
**b)** 14
**c)** 12
**d)** 10
**e)** 9

29. What is the probability of rolling a red die and a blue die and having the number showing in the red die to be larger than the number showing in the blue one?

**a)** $\frac{4}{9}$
**b)** $\frac{1}{2}$
**c)** $\frac{19}{36}$
**d)** $\frac{2}{3}$
**e)** $\frac{5}{12}$

30. If $f(x)$ is a function that satisfies $f(2x + 1) = 2f(x) + 1$ for all $x$, and if $f(0) = 2$, then $f(3) =$

**a)** 5
**b)** 9
**c)** 11
**d)** 13
**e)** *None of the above*

31. If the line $y = 2x + b$ is tangent to the circle $x^2 + y^2 = 1$, the possible values of $b$ are

**a)** $\pm\sqrt{5}$
**b)** $\pm\sqrt{2}$
**c)** $\pm\frac{\sqrt{5}}{2}$
**d)** $\pm\frac{\sqrt{2}}{2}$
**e)** *None of the above*

32. Let

$$A = 2008 + \frac{1}{2008} \qquad B = 2008 + \frac{1}{2008 + \frac{1}{2008}} \qquad C = 2008 + \frac{1}{2008 + \frac{1}{2008 + \frac{1}{2008}}}$$

The number $A, B$, and $C$ arranged in increasing order are

**a)** $C, A, B$
**b)** $A, B, C$
**c)** $B, C, A$
**d)** $C, B, A$
**e)** $B, A, C$

33. Assume that $b$ and $c$ are two integers that are greater than one. In base $b$, $c^2$ is written as 10. Then $b^2$, when written in base $c$ is

**a)** 100
**b)** 101
**c)** 10000
**d)** 1010
**e)** *It cannot be determined*

34. The largest solution of $2\log_{10} x = \log_{10}(3x - 20) + 1$ is

**a)** 10
**b)** $\frac{20}{3}$
**c)** $\frac{23}{3}$
**d)** 100
**e)** 20

35. The only value of $a$ for which the simultaneous equations

$$\begin{aligned} 2x + 3y &= 5 \\ x + ay &= 2 \end{aligned}$$

would have no solutions is

**a)** 0
**b)** 2
**c)** $\frac{3}{2}$
**d)** $\frac{1}{2}$
**e)** $-\frac{1}{2}$

36. How many subsets of $\{a, b, c, d, e, f, g\}$ contain both $a$ and $b$?

**a)** 32
**b)** 25
**c)** 16
**d)** 12
**e)** 9

37. If $xy = 2$ and $x^2 + y^2 = 5$, then

$$\frac{x}{y} + \frac{y}{x} =$$

**a)** 0
**b)** 1
**c)** 25
**d)** $\frac{5}{2}$
**e)** $\frac{2}{5}$

38. If $x < -2$, then $|1 - |1 - x||$ equals

**a)** $2 + x$
**b)** $-x$
**c)** $x$
**d)** $-2 - x$
**e)** $-2$

39. If $a$ and $b$ are integers such that $x^2 - x - 1$ is a factor of $ax^3 + bx^2 + 1$, then $b$ is equal to

**a)** 2
**b)** $-2$
**c)** $-1$
**d)** 1
**e)** 0

40. If $\log_{\sin x}(\cos x) = \frac{1}{2}$, then $\sin x =$

**a)** $\frac{2\sqrt{3}-1}{2}$
**b)** $\frac{2}{\sqrt{3}}$
**c)** $\frac{1}{\sqrt{3}}$
**d)** $\frac{\sqrt{5}+1}{3}$
**e)** $\frac{\sqrt{5}-1}{2}$

CSU FRESNO MATHEMATICS FIELD DAY
APRIL 19, 2008
MAD HATTER MARATHON 11-12
PART II

1. Let $f(x)$ be a quadratic polynomial such that $f(-2) = -3$ and $f(2) = 21$. Then the coefficient of $x$ in $f(x)$ is

   **a)** 4
   **b)** $-4$
   **c)** 6
   **d)** $-6$
   **e)** 0

2. Sam and Susie are brother and sister. Sam has twice as many sisters as brothers. Susie has twice as many brothers as sisters. The number of girls in the family is

   **a)** 2
   **b)** 1
   **c)** 3
   **d)** 4
   **e)** 5

3. If $x, y \in \mathbb{R}$, and $x^2 + y^2 = 1$, then the maximum value of $(x + y)^2$ is

   **a)** $\frac{3}{2}$
   **b)** $\sqrt{5}$
   **c)** 3
   **d)** 2
   **e)** 1

4. When the base of a triangle is decreased 10% and the altitude is increased 10%, then the area is

   **a)** *Unchanged*
   **b)** *increased* 10%
   **c)** *decreased* 10%
   **d)** *increased* 1%
   **e)** *decreased* 1%

5. The largest integer $n$ such that $2^n$ divides $17^9 - 9^9$ is

   **a)** 7
   **b)** 6
   **c)** 5
   **d)** 4
   **e)** 3

6. If John gets a 97 on his next math test, his average test score will be 90. If he gets 73, his average will be 87. How many tests has John already taken?

   **a)** 3
   **b)** 4
   **c)** 5
   **d)** 6
   **e)** 7

7. Suppose the number of elements in the a $S$ is 105 and that $S$ is partitioned into $n$ subsets of $11m + 2$ elements each. Then $m$ is

**a)** 6
**b)** 5
**c)** 4
**d)** 3
**e)** 2

8. If it were two hours later, it would be half as long until midnight as it would be if it were one hour later. The time now is
   **a)** $10:00\ PM$
   **b)** $9:30\ PM$
   **c)** $9:00\ PM$
   **d)** $8:00\ PM$
   **e)** $7:00\ PM$

9. A pair of dice is thrown. What is the probability that the two numbers that appear differ by exactly 2?
   **a)** $\frac{2}{3}$
   **b)** $\frac{1}{3}$
   **c)** $\frac{1}{6}$
   **d)** $\frac{2}{9}$
   **e)** $\frac{1}{9}$

10. Two successive discounts of 10% have the same effect as a single discount of
    **a)** 21%
    **b)** 20%
    **c)** 19%
    **d)** 18%

11. The number of **even** positive integers that are divisors of $720 = 2^4 \cdot 3^2 \cdot 5$ is
    **a)** 15
    **b)** 16
    **c)** 24
    **d)** 25
    **e)** 29

12. Let $x$ be the smallest number larger than 2 that leaves a remainder of 2 when divided by $3, 5$, and 6. Then the sum of the digits of $x$ is
    **a)** 1
    **b)** 5
    **c)** 11
    **d)** 12
    **e)** 14

13. If the width of a particular rectangle is doubled and the length is increased by three, then the area is tripled. What is the length of the rectangle?
    **a)** 1
    **b)** 2
    **c)** 3
    **d)** 6
    **e)** 9

14. For each positive integer $n$, define
$$S_n = 1^4 + 2^4 + \cdots + n^4$$
    What is the value of $\log_{10}(S_{100} - S_{99})$?

**a)** 4
**b)** 8
**c)** 9
**d)** 10
**e)** 100

15. In how many ways can you walk up a stairway with 6 steps if you can take one or two steps at a time?

**a)** 9
**b)** 10
**c)** 11
**d)** 12
**e)** 13

16. Find the number of times between 1 o'clock and 4 o'clock when the hour hand is perpendicular to the minute hand

**a)** 3
**b)** 4
**c)** 5
**d)** 6

17. If $A + B = 12$, $B + C = 10$, $C + D = 16$, then $A + D =$?

**a)** 20
**b)** 18
**c)** 12
**d)** 14
**e)** 16

18. Which of the following numbers is a solution of the equation

$$(x^2 - 2)(x^2 + 6)(x^3 - 8) = 0 \ ?$$

**a)** 5
**b)** 4
**c)** 3
**d)** 2
**e)** 1

19. At a party, every two people shook hands once. How many people attended the party if there were exactly 66 handshakes?

**a)** 65
**b)** 54
**c)** 33
**d)** 22
**e)** 12

20. Suppose that $x$ is a complex number such that $x^2 - x + 1 = 0$. What is the value of $x^3$?

**a)** 0
**b)** 1
**c)** $-1$
**d)** 1.5
**e)** 2

21. At how many points does the graph of

$$y = (x - 2)(2x^2 - 5x + 4)(2x^2 - 7x + 4)$$

intersect the $x$-axis?

**a)** 0
**b)** 1
**c)** 2
**d)** 3
**e)** 5

22. Which of the following numbers is the largest?

**a)** $10^{1000}$
**b)** $1000^{100}$
**c)** $5^{4000}$
**d)** $3^{2000}$
**e)** $2^{10000}$

23. How many integers $1 \leq x \leq 100$ are there such that $x^2 + x^3$ is the square of an integer?

**a)** 6
**b)** 7
**c)** 8
**d)** 9
**e)** 10

24. Suppose we draw 100 horizontal lines and 100 vertical lines in the plane. How many "pieces" of the plane are formed by cutting along all of these lines? Note that some of the pieces may have infinite area.

**a)** 10000
**b)** 10001
**c)** 10004
**d)** 10201
**e)** 10204

25. A used car dealer sold two cars and received \$560 for each car. One of these transactions amounted to a 40% profit for the dealer, whereas the other amounted to a 20% loss. What is the dealers net profit on the two transactions?

**a)** \$112
**b)** \$84
**c)** \$56
**d)** \$36
**e)** \$20

26. How many integers between 100 and 1000 are multiples of 7?

**a)** 128
**b)** 130
**c)** 132
**d)** 134
**e)** 136

27. Given that the vertex of the parabola $y = x^2 + 8x + k$ is on the $x$-axis, what is the value of $k$?

**a)** 4
**b)** 8
**c)** 16
**d)** 24
**e)** 0

28. What is the value of $\log_2(\log_2(\log_2 16))$ ?

**a)** 1
**b)** 2
**c)** 4
**d)** 8
**e)** 0

29. A square has perimeter $p$ and area $A$. If $A = 2p$, then what is the value of $p$?

**a)** 54
**b)** 48
**c)** 36
**d)** 32
**e)** 24

30. There are 29 people in a room. Of these, 11 speak French, 24 speak English and 3 speak neither French nor English. How many people in the room speak both French and English?

**a)** 3
**b)** 4
**c)** 6
**d)** 8
**e)** 9

31. A line through the points $(m, -9)$ and $(7, m)$ has slope $m$. What is the value of $m$?

**a)** 1
**b)** 2
**c)** 3
**d)** 4
**e)** 5

32. Which of the following numbers is equal to the sum

$$8^8 + 8^8 + 8^8 + 8^8 + 8^8 + 8^8 + 8^8 + 8^8 \quad ?$$

**a)** $8^9$
**b)** $8^{8^8}$
**c)** $64^8$
**d)** $8^{64}$
**e)** $64^{64}$

33. Two perpendicular lines, intersecting at the center of a circle of radius 1, divide the circe into four parts. A smaller circle is inscribed in one of those parts. What is the radius of the smaller circle?

**a)** $\frac{1}{3}$
**b)** $\frac{2}{5}$
**c)** $\frac{1}{2}$
**d)** $\sqrt{2} - 1$
**e)** $2 - \sqrt{2}$

34. If $2^4 \cdot 3^8 = n \cdot 6^4$, then $n =$

**a)** 12
**b)** 24
**c)** 27
**d)** 54
**e)** 81

35. The graphs of the lines $y = x - 2$ and $y = mx + 3$ intersect at a point whose $x$-coordinate and $y$-coordinate are both positive if and only if

**a)** $m < 1$
**b)** $m = 1$
**c)** $-\frac{3}{2} < m < 0$
**d)** $-\frac{3}{2} < m$
**e)** $-\frac{3}{2} < m < 1$

36. For each real number we define $[\alpha]$ to be the greatest integer which is less than or equal to $\alpha$. If $x$ and $y$ are real numbers such that $[\sqrt{x}] = 9$ and $[\sqrt{y}] = 12$, then the largest possible value of $[x + y]$ is

**a)** 225
**b)** 242
**c)** 256
**d)** 268
**e)** 270

37. The volume of a large (solid) cube is 125 cubic inches. A new shape is formed by removing a 1 in $\times$ 1 in $\times$ 1 in cube from one corner of the large cube. The surface area of this new shape in square inches is

**a)** 250
**d)** 225
**c)** 180
**d)** 150
**e)** 120

38. Which of the following shapes has the largest area?

**a)** *A right triangle with legs of length* 6 *and* 8, *and hypothenuse of length* 10
**b)** *A square with side of length* 5
**c)** *A circle with radius of length* 3
**d)** *A rectangle with sides of length* 3 *and* 9

39. The length of the shorter side of a rectangle is 2 units. The length of each diagonal is 4 units. What is the acute angle between the diagonals ?

**a)** $15^o$
**b)** $22.5^o$
**c)** $45^o$
**d)** $60^o$
**e)** $75^o$

40. For how many positive integers $n$ does 2008 have remainder of 8 when divided by $n$ ?

**a)** 7
**b)** 9
**c)** 11
**d)** 15
**e)** 17

MAD HATTER MARATHON 11-12

## PART I ANSWERS

| | |
|---|---|
| 1.$D$ | 21.$C$ |
| 2.$B$ | 22.$C$ |
| 3.$C$ | 23.$C$ |
| 4.$D$ | 24.$B$ |
| 5.$E$ | 25.$E$ |
| 6.$A$ | 26.$C$ |
| 7.$C$ | 27.$B$ |
| 8.$A$ | 28.$D$ |
| 9.$D$ | 29.$E$ |
| 10.$E$ | 30.$C$ |
| 11.$D$ | 31.$A$ |
| 12.$E$ | 32.$C$ |
| 13.$D$ | 33.$C$ |
| 14.$D$ | 34.$E$ |
| 15.$C$ | 35.$C$ |
| 16.$C$ | 36.$A$ |
| 17.$A$ | 37.$D$ |
| 18.$C$ | 38.$B$ |
| 19.$D$ | 39.$B$ |
| 20.$C$ | 40.$E$ |

## PART II ANSWERS

| | |
|---|---|
| 1.$C$ | 21.$D$ |
| 2.$A$ | 22.$E$ |
| 2.$D$ | 23.$D$ |
| 4.$E$ | 24.$D$ |
| 5.$E$ | 25.$E$ |
| 6.$E$ | 26.$A$ |
| 7.$D$ | 27.$C$ |
| 8.$C$ | 28.$A$ |
| 9.$D$ | 29.$D$ |
| 10.$C$ | 30.$E$ |
| 11.$C$ | 31.$C$ |
| 12.$B$ | 32.$A$ |
| 13.$D$ | 33.$D$ |
| 14.$B$ | 34.$E$ |
| 15.$E$ | 35.$E$ |
| 16.$C$ | 36.$D$ |
| 17.$B$ | 37.$D$ |
| 18.$D$ | 38.$C$ |
| 19.$E$ | 39.$D$ |
| 20.$C$ | 40.$D$ |