

CSU FRESNO MATHEMATICS FIELD DAY  
APRIL 22, 2006  
MAD HATTER MARATHON 9-10  
PART I

1. Find the coefficient of  $x^4y^2$  in  $(2x - y)^6$ .
  - (a) 64
  - (b) -198
  - (c) 240
  - (d) -160
  
2. A ladder leaning against a vertical wall makes an angle of  $30^\circ$  with the wall. If the foot of the ladder is 3 feet from the wall, give the length of the ladder and the height it extends up the wall.
  - (a) 6 feet long,  $3\sqrt{3}$  feet up the wall
  - (b) 1.5 feet long,  $\frac{3\sqrt{3}}{4}$  feet up the wall
  - (c) 6 feet long,  $\frac{3\sqrt{3}}{4}$  feet up the wall
  - (d) 1.5 feet long,  $3\sqrt{3}$  feet up the wall
  
3. Determine the sum  $3 + 7 + 11 + \dots + 35$ .
  - (a) 140
  - (b) 171
  - (c) 315
  - (d) 342
  
4. Tom walks at a rate of 4 miles per hour and Dick walks at a rate of 3 miles per hour. If Dick has a half hour head start, how long will it take for Tom to catch up with him?
  - (a) a half hour
  - (b) one hour
  - (c) one and a half hours
  - (d) two hours
  
5. What is  $\sqrt{30}$  to the nearest hundredth?
  - (a) 5.25
  - (b) 5.38
  - (c) 5.48
  - (d) 5.77
  
6. If we divide an regular polygon with  $x$  sides into  $x$  equilateral triangles, what fraction of a right angle is an angle of one of the  $x$  triangles?

- (a)  $\frac{2x-8}{x}$
- (b)  $\frac{2x+2}{x}$
- (c)  $\frac{2x+4}{x}$
- (d) none of the above

7. Find the first of three consecutive odd integers whose sum is 63.

- (a) 15
- (b) 17
- (c) 19
- (d) 21

8. A rectangular field is 6 m narrower than it is long. If its width is reduced by 2 m and its length increased by 3 m, its area is unchanged. Find the length of the field.

- (a) 24 m
- (b) 18 m
- (c) 12 m
- (d) none of the above

9. Determine the solution of the inequality

$$5(y - 2) - 6y \leq 3(5y + 2).$$

- (a)  $y \geq \frac{1}{4}$
- (b)  $y \leq \frac{8}{7}$
- (c)  $y \leq -1$
- (d)  $y \geq -1$

10. A bowler scores 146, 132, 153, 148, and 138 in five games. What must be bowled in the next game to attain an average of 150?

- (a) 190
- (b) 183
- (c) 160
- (d) 153

11. Evaluate

$$2 + 8 \times 3 - 4 \div 2 + 5.$$

- (a) 33
- (b) 29
- (c) 18
- (d) 16

12. A chemist has two solutions of hydrochloric acid. One is a 40% solution and the other is a 90% solution. How many liters of each should she mix to get 10 liters of a 50% solution?
- (a) 8 liters of 40% and 2 liters of 90%
  - (b) 7 liters of 40% and 3 liters of 90%
  - (c) 6 liters of 40% and 4 liters of 90%
  - (d) 9 liters of 40% and 1 liter of 90%
13. Evaluate  $i^{35}$ .
- (a) 1
  - (b) -1
  - (c)  $i$
  - (d)  $-i$
14. Simplify  $(-8x^4y^{-3})^{\frac{1}{3}}$
- (a)  $\frac{-2x^{\frac{4}{3}}}{y}$
  - (b)  $2ix^{\frac{4}{3}}y^{-1}$
  - (c)  $-2x^{\frac{13}{3}}y^{-\frac{8}{3}}$
  - (d)  $-2ix^{\frac{13}{3}}y^{-\frac{8}{3}}$
15. Evaluate the sum  $2 + 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots$
- (a)  $\frac{7}{3}$
  - (b) 4
  - (c) 5
  - (d) 6
16. Determine  $x$  if  $\ln 2 - \frac{1}{3} \ln 3 + 4 \ln e = 2 \ln x$ .
- (a)  $e^2 3^{\frac{1}{6}} \sqrt{2}$
  - (b)  $\frac{1}{2} \frac{e^4}{3^{\frac{1}{6}}}$
  - (c)  $\frac{e^2 \sqrt{2}}{3^{\frac{1}{6}}}$
  - (d)  $\pm \frac{e^2 \sqrt{2}}{3^{\frac{1}{6}}}$
17. Coming home from a meeting, Rick gets a ride from a friend part of the way and walks the remainder of the way. Rick's friend drives an average of 55 miles per hour and Rick walks an average of 4 miles per hour. If it takes Rick 5 hours to travel the 173 miles home, how far did he walk?
- (a) 4 miles
  - (b) 8 miles

- (c) 12 miles
- (d) 16 miles

18. Evaluate

$$\frac{4351^2 - 4347^2}{(4350)(4353) - 4351^2}.$$

- (a)  $\frac{1}{2}$
  - (b) 4
  - (c) 8
  - (d) none of the above
19. How many posts are needed to fence a rectangular plot of land that is 40 m wide and  $127\frac{1}{2}$  m long if the posts will be set  $2\frac{1}{2}$  m apart (from center of post to center of post)?
- (a) 133
  - (b) 134
  - (c) 136
  - (d) 138

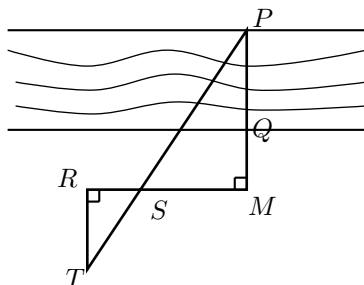
20. Simplify

$$1 - \frac{\frac{1}{x-1} + \frac{x}{x+1}}{\frac{1}{x+1} + \frac{x}{1-x}}.$$

- (a) 2
  - (b)  $x^2$
  - (c)  $\frac{x^2-2}{x^2-1}$
  - (d)  $\frac{x^2}{1-x^2}$
21. Evaluate

$$\frac{4}{4 + \frac{4}{4 + \frac{4}{4 + \frac{4}{4}}}}.$$

- (a)  $\frac{1}{4}$
  - (b)  $\frac{4}{11}$
  - (c)  $\frac{44}{53}$
  - (d) 1
22. Suppose you wish to measure the distance across the river shown below. You find that the distance SM is 35 feet, RS is 8 feet, RT is 16 feet, and QM is 10 feet. How far is it across the river?



- (a) 40 feet  
 (b) 50 feet  
 (c) 70 feet  
 (d) 60 feet
23. What is the range of the function  $y = |x|$ ?
- (a)  $x \geq 0$   
 (b)  $-\infty < x < \infty$   
 (c)  $y \geq 0$   
 (d)  $-\infty < y < \infty$
24. The stiffness of a rectangular beam is directly proportional to the product of the breadth and depth and is inversely proportional to the square of the length. Letting  $b$  be breadth of the beam,  $d$  be the depth of the beam, and  $l$  be the length of the beam, which of the following represents  $S$ , the stiffness of the beam?
- (a)  $S = k \frac{bd}{l^2}$   
 (b)  $S = k b d l^2$   
 (c)  $S = k \frac{l^2}{bd}$   
 (d)  $S = \frac{bd}{l^2}$
25. If  $x$  is directly proportional to the square of  $y$  and inversely proportional to  $z$ , find  $x$  when  $y = 9$  and  $z = 27$  if  $x = 8$  when  $y = 4$  and  $z = 6$ .
- (a)  $\frac{3}{4}$   
 (b)  $\frac{3}{2}$   
 (c) 1  
 (d) 9
26. Compute  $(2 + 3i)(1 - 2i)$ .

- (a)  $-4 - i$   
(b)  $8 - i$   
(c)  $8 + i$   
(d)  $-4 + i$
27. The longest rod that will just fit inside a square box, if placed diagonally top to bottom, is 16 inches. How long is one side of the box?
- (a)  $\frac{16}{\sqrt{3}}$   
(b)  $\frac{16}{\sqrt{2}}$   
(c)  $\frac{16}{3}$   
(d) 2
28. If  $w : h$  equals  $h : (2h - w)$  and  $w = 5$ , what is  $h$ ?
- (a) -45  
(b)  $\pm\sqrt{15}i$   
(c) 5  
(d) none of the above
29. One cubical box is packed inside of another box whose side is 2 meters longer. If it takes 56 cubic meters of packing material to fill the space around the smaller carton, what is the volume of the larger box?
- (a) 4 cubic meters  
(b) 8 cubic meters  
(c) 64 cubic meters  
(d) none of the above
30. Solve  $\sqrt{x - 2} = 2 - x$ .
- (a)  $x = 3$   
(b)  $x = 2$   
(c)  $x = 2$  or  $x = 3$   
(d) No solution
31. Which of the following is the equation of a parabola pointing upward?
- (a)  $y = x^2 - 4x + 8$   
(b)  $y^2 - 2y + 1 = x$   
(c)  $y = -x^2 - 3x + 9$   
(d)  $x^2 + 4y^2 = 1$
32. Convert 384 (base 10) to a base hexadecimal (base 16) number.

- (a)  $100_{16}$   
(b)  $120_{16}$   
(c)  $140_{16}$   
(d)  $180_{16}$
33. Convert  $128_{16}$  to a base 10 number.  
(a) 4736  
(b) 200  
(c) 256  
(d) 296
34. Evaluate  $\binom{5}{3} + \binom{6}{2}$ .  
(a) 380  
(b) 90  
(c) 25  
(d) 11
35. What is the sum of the coefficients of the expansion of  $(2a + b - c)^8$ ?  
(a) 720  
(b) 676  
(c) 512  
(d) 256
36. Write  $0.03\bar{1}$  as a fraction in lowest terms.  
(a)  $\frac{7}{225}$   
(b)  $\frac{1}{225}$   
(c)  $\frac{14}{45}$   
(d)  $\frac{2}{45}$
37. Find the first term in the arithmetic progression whose sixth term is 13 and whose twelfth term is 25.  
(a) 4  
(b) 3  
(c) 2  
(d) 1
38. How many different total scores can be obtained from three tosses of a die?  
(a)  $6^3$

(b)  $(6)(5)(4)$

(c) 16

(d) 6

39. Find the  $x$ -intercepts of  $y = x^4 - 8x^2 + 16$ .

(a)  $x = 2$

(b)  $x = -2$

(c)  $x = 16$

(d) Both a) and b) are  $x$ -intercepts.

40. A circle with radius  $r$  has area  $A$ . If the area of the circle has doubled, what is its radius in terms of  $r$ .

(a)  $2r$

(b)  $4r$

(c)  $\sqrt{2}r$

(d)  $\frac{r}{\sqrt{2}}$

MAD HATTER MARATHON 9-10  
PART I ANSWERS

1. c
2. a
3. b
4. c
5. c
6. a
7. c
8. a
9. d
10. b
11. b
12. a
13. d
14. a
15. b
16. c
17. b
18. c
19. b
20. a
21. c
22. d
23. c
24. a
25. d
26. b

27. a

28. c

29. c

30. b

31. a

32. d

33. d

34. c

35. d

36. a

37. b

38. c

39. d

40. c

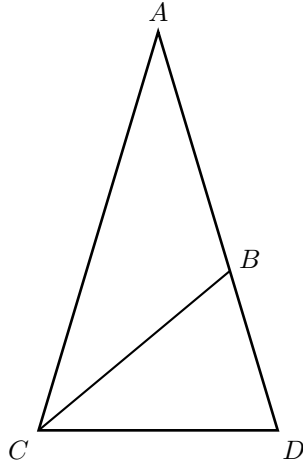
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PART II

1. How many different 7-character license plates are possible if the first two characters are letters and the remaining five characters are numbers?
  - (a)  $26^2 10^5$
  - (b)  $26^2 9^5$
  - (c)  $26 \times 25 \times 10 \times 9 \times 8 \times 7 \times 6$
  - (d)  $26 \times 25 \times 9 \times 8 \times 7 \times 6 \times 5$
  
2. If  $f(x) = \sqrt{4-x}$  and  $g(x) = x^2$ . What is  $f(g(3))$ ?
  - (a) 1
  - (b) -5
  - (c) 5
  - (d)  $(f \circ g)(3)$  does not exist.
  
3. If  $x = \frac{2}{3+4y}$ , find  $y$ .
  - (a)  $\frac{3+4x}{2}$
  - (b)  $\frac{3}{2} + 2x$
  - (c)  $\frac{1}{2x} - \frac{3}{4}$
  - (d) none of the above
  
4. Find the equation of the line passing through the point  $(-2, 4)$  and perpendicular to the line  $x + 2y = 17$ .
  - (a)  $y = 2x + 8$
  - (b)  $y = -\frac{1}{2}x + 2$
  - (c)  $y = -2x$
  - (d)  $y = 2x$
  
5. Solve the inequality  $\frac{2}{|3x-4|} < 1$ .
  - (a)  $x > \frac{2}{3}$
  - (b)  $x > 2$
  - (c)  $x < \frac{2}{3}$
  - (d)  $x < \frac{2}{3}$  or  $x > 2$
  
6. Solve the inequality  $x^2 - 3x + 2 < 0$ .
  - (a)  $x < 1$

- (b)  $x > 2$   
 (c)  $1 < x < 2$   
 (d) none of the above
7. To get to the post office from his house, Sam walks 5 miles East, then 3 miles North, and then 1 mile West. What is the actual distance from Sam's house to the post office?
- (a) 9 miles  
 (b) 8 miles  
 (c) 5 miles  
 (d) none of the above
8. Simplify and write the answer in scientific notation  $\frac{72,000 \times 81 \times 10^6}{4 \times 10^{-7}}$ .
- (a)  $1.458 \times 10^{19}$   
 (b)  $1.458 \times 10^{18}$   
 (c)  $1.458 \times 10^{17}$   
 (d)  $1.458 \times 10^{13}$
9. Simplify the expression  $\left(16a^{\frac{4}{3}}b^{\frac{1}{2}}\right)^{-\frac{1}{2}}$ .
- (a)  $-4a^{\frac{5}{6}}$   
 (b)  $\frac{1}{4}a^{-\frac{2}{3}}b^{-\frac{1}{4}}$   
 (c)  $\frac{1}{4}a^{\frac{3}{2}}b^{-1}$   
 (d)  $4a^{\frac{3}{2}}b$
10. Evaluate  $\frac{10!}{5!5!}$ .
- (a) 253  
 (b) 126  
 (c) 20  
 (d) none of the above
11. A retail establishment accepts either the MasterCard or VISA credit cards. A total of 24% of its customers carry a MasterCard, 61% carry a VISA card, and 11% carry both. What percentage of its customers carry a credit card that the establishment will accept?
- (a) 96%  
 (b) 85%  
 (c) 74%  
 (d) none of the above
12. What is the greatest common divisor of 1776 and 1976?

- (a) 16  
 (b) 8  
 (c) 4  
 (d) 2
13. What is the least common multiple of 36 and 243?
- (a) 8748  
 (b) 2916  
 (c) 972  
 (d) 108
14. Simplify  $\frac{-5}{\sqrt[3]{2}-\sqrt[3]{7}}$ .
- (a)  $\sqrt[3]{4} + \sqrt[3]{106} + \sqrt[3]{49}$   
 (b)  $\sqrt[3]{2} + \sqrt[3]{7}$   
 (c)  $\sqrt[3]{4} + \sqrt[3]{14} + \sqrt[3]{49}$   
 (d)  $\sqrt[3]{4} + \sqrt[3]{49}$
15. Solve  $\sqrt{x+12} + \sqrt{x} = 2$ .
- (a) -4  
 (b) 0  
 (c) 4  
 (d) no solution
16. The circumference of the front tires of a sport car is 2 feet more than the circumference of the rear tires. If the rear wheels make 22 revolutions in traveling 1 mile, find the radius of the front tires.
- (a)  $\frac{60}{\pi}$  feet  
 (b)  $\frac{61}{\pi}$  feet  
 (c)  $\frac{120}{\pi}$  feet  
 (d)  $\frac{121}{\pi}$  feet
17. If, in a triangle  $ABC$ , median  $BD$  is such that  $m\angle A = m\angle DBC$ , and  $m\angle ADB = 45^\circ$ , find  $m\angle A$ .
- (a)  $30^\circ$   
 (b)  $45^\circ$   
 (c)  $60^\circ$   
 (d) none of the above
18. The operand  $*$  indicates that one should double the difference between  $x$  and 2, and the operand  $\#$  indicates that one should take the difference of twice  $x$  and 2. What is  $\# - *$ ?

- (a) -2  
(b) 0  
(c) 2  
(d) not enough information
19. If an arc of  $45^\circ$  on circle  $A$  has the same length as an arc of  $30^\circ$  on circle  $B$ , then the ratio of the area of circle  $A$  to the area of circle  $B$  is
- (a)  $\frac{9}{4}$   
(b)  $\frac{3}{2}$   
(c)  $\frac{2}{3}$   
(d)  $\frac{4}{9}$
20. The number of inches in the perimeter of an equilateral triangle is equal to the number of square inches in the area of its circumscribed circle. What is the radius, in inches, of the circle?
- (a)  $\frac{3\sqrt{2}}{\pi}$   
(b)  $\frac{3\sqrt{3}}{\pi}$   
(c)  $\sqrt{3}$   
(d)  $\sqrt{3}\pi$
21. For how many positive integers  $m$  does there exist at least one integer  $n$  such that  $mn \leq m+n$ ?
- (a) 4  
(b) 6  
(c) 12  
(d) infinitely many
22. All students in a school were given a questionnaire regarding a movie just shown in the auditorium. Fifty-five percent of the students answered the questionnaire, and of that group, 41% liked the movie. What is the maximum percentage of students in the school who could like the movie?
- (a) 22.55%  
(b) 67.55%  
(c) 22.45%  
(d) 67.45%
23. Observe triangle  $ABC$  in the figure below. Segment  $AB$  is congruent to segment  $BC$ , and  $BC$  is congruent to segment  $CD$ . If segment  $BC$  bisects angle  $ACD$ , find  $m\angle CAD$ .



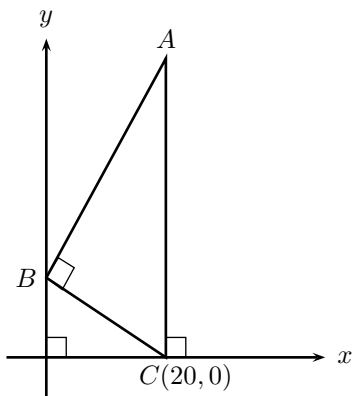
- (a)  $30^\circ$   
 (b)  $36^\circ$   
 (c)  $45^\circ$   
 (d)  $60^\circ$
24. If  $a$ ,  $b$ ,  $a + b$ , and  $a - b$  are all prime numbers, which of the following statements must be true about the sum of these four numbers?
- (a) The sum is odd and prime.  
 (b) The sum is odd and divisible by 3.  
 (c) The sum is odd and divisible by 7.  
 (d) The sum is even.
25. Kyle says that there is a 55% chance that he will go to the library tomorrow if it is raining at noon and a 30% chance if it is not raining at noon. The weather person predicts a 40% chance of rain at noon. On the basis of the information given, what is the chance that Kyle will go to the library tomorrow?
- (a) 51%  
 (b) 40%  
 (c) 33%  
 (d) 22%
26. If 26 blops weigh as much as 4 glorps and 2 gloops, while 8 blops and 2 glorps have the same weight as 2 gloops, how many blops have the same weight as 3 gloops?
- (a) 10  
 (b) 12  
 (c) 21  
 (d) none of the above
27. What is the smallest positive prime  $p$  greater than 2 such that  $p^3 + 7p^2$  is a perfect square?

- (a) 13
- (b) 17
- (c) 23
- (d) 29

28. The set of real numbers  $a$  such that the equation  $x^3 - ax^2 - 2ax + a^2 - 1 = 0$  has **exactly** one real solution in  $x$  satisfies:

- (a)  $a > -\frac{3}{4}$
- (b)  $a \geq -\frac{3}{4}$
- (c)  $a < \frac{3}{4}$
- (d)  $a \leq \frac{3}{4}$

29. In the figure below, if  $\frac{AB}{BC} = \frac{4}{3}$ , what is the  $y$ -coordinate of point  $B$ ?



- (a) 5
  - (b) 10
  - (c) 15
  - (d) none of the above
30. Melinda's lights went out. She has 3 pairs of red socks, 2 pairs of white socks, and 5 pairs of blue socks in her drawer. How many pairs of socks must she remove in order to guarantee that she has one pair of each color?
- (a) 3
  - (b) 5
  - (c) 7
  - (d) 9
31. Donald Duck can eat 2 pizzas in 3 minutes, while Goofy can eat 3 pizzas in 2 minutes. At these rates, how many pizzas can they eat together in an hour?

- (a) 54
  - (b) 96
  - (c) 130
  - (d) 210
32. Suppose a positive integer  $N$  is divisible by both 9 and 21. What is the smallest possible number of positive integers that divide  $N$ ?
- (a) 6
  - (b) 5
  - (c) 4
  - (d) 3
33. Which of the following is the largest number?
- (a)  $2^{(3^4)}$
  - (b)  $4^{(3^2)}$
  - (c)  $8^{(4^2)}$
  - (d)  $(16^8)^2$
34. Rank the following collection of round pizzas in increasing order of their area: (A) five 10-inch pizzas; (B) three 14-inch pizzas; (C) two 16-inch pizzas
- (a) ABC
  - (b) ACB
  - (c) BCA
  - (d) CAB
35. Let  $a < b < c$  be the lengths of the sides of triangle  $ABC$ . If  $a^2 + b^2 < c^2$ , which of the following must be true?
- (a) The angles of  $ABC$  are all acute.
  - (b) Some angle of  $ABC$  is obtuse.
  - (c)  $ABC$  is equilateral.
  - (d) No such triangle can exist.
36. If three distinct counting numbers have a sum of 10 and a product of 20, what is the median of the three numbers?
- (a) 3
  - (b) 4
  - (c) 5
  - (d) There is not enough information given.

37. Let  $ABC$  be an equilateral triangle with sides of length  $x$ . Let  $P$  be the point of intersection of the three angle bisectors. Find  $\overline{AP}$ .
- (a)  $\frac{x\sqrt{3}}{3}$
  - (b)  $\frac{x\sqrt{3}}{6}$
  - (c)  $\frac{5x\sqrt{3}}{6}$
  - (d)  $\frac{2x\sqrt{3}}{3}$
38. How many 2-digit numbers,  $n \geq 10$  are there such that both digits are squares (e.g., 10 and 41 are two such numbers)?
- (a) 3
  - (b) 6
  - (c) 8
  - (d) 12
39. An ancient manuscript had the statement (translated to present-day notation) that  $x^2+bx+30$  has two integer roots. Unfortunately, it was impossible to read the positive integer  $b$ . How many possibilities are there for  $b$ ?
- (a) none
  - (b) 3
  - (c) 4
  - (d) 6
40. If 7 dogs can eat 7 bowls of dog food in 7 minutes, how long will it take 14 dogs to eat 14 bowls of dog food?
- (a) 28 minutes
  - (b) 21 minutes
  - (c) 14 minutes
  - (d) 7 minutes

MAD HATTER MARATHON 9-10  
PART II ANSWERS

1. a
2. d
3. c
4. a
5. d
6. c
7. c
8. a
9. b
10. b
11. c
12. b
13. c
14. c
15. d
16. d
17. a
18. c
19. d
20. b
21. d
22. b
23. b
24. a
25. b
26. c

27. d

28. c

29. c

30. d

31. c

32. a

33. a

34. b

35. b

36. b

37. a

38. d

39. c

40. d