

CSU FRESNO MATHEMATICS FIELD DAY
APRIL 19, 2008
MAD HATTER MARATHON 9-10
PART I

1. What is

$$\frac{5}{6^{-2}8^{\frac{1}{3}}}$$

- (a) 60
 - (b) 70
 - (c) 80
 - (d) 90
2. A jar contains 3 red marbles, 4 blue marbles, and 5 green marbles. Janice takes one marble out of the jar, and then Tom takes one marble out of the same jar. What is the probability that Janice drew a red marble and Tom drew a green marble?
- (a) $\frac{5}{44}$
 - (b) $\frac{5}{48}$
 - (c) $\frac{5}{33}$
 - (d) $\frac{5}{36}$
3. What is the sum of all of the even numbers between 1 and 25?
- (a) 338
 - (b) 312
 - (c) 169
 - (d) 156
4. In the hexadecimal number system, what is $1A+2E$?
- (a) 26
 - (b) 38
 - (c) 48
 - (d) 72
5. Fran uses cubes of 1 cubic inch (unit cubes) to create a cube that is 64 cubic inches in volume. If she paints all faces of the larger cube blue, and then breaks the cube apart into the unit cubes, how many of the unit cubes have no paint on them?
- (a) 24
 - (b) 8
 - (c) 6
 - (d) 4
6. What angle between two equal sides of a triangle will result in an isosceles triangle with the largest area?
- (a) 90°
 - (b) 60°
 - (c) 45°

- (d) 30°
7. Find a number such that 6 more than one half the number is twice the number.
- (a) -6
(b) -4
(c) 4
(d) 6
8. Which of the following represents the number 34 (base 10) as a base-6 number?
- (a) 100_6
(b) 54_6
(c) 34_6
(d) none of the above
9. What nonzero real value of x satisfies the equation $(7x)^{14} = (14x)^7$?
- (a) $\frac{1}{7}$
(b) $\frac{2}{7}$
(c) 1
(d) 7
10. In a triangle with integer side lengths, one side is three times as long as a second side and the length of the third side is 15. What is the largest possible perimeter of the triangle?
- (a) 45
(b) 44
(c) 43
(d) 42
11. What is the maximum number of possible points of intersection of a circle and a triangle?
- (a) 3
(b) 4
(c) 5
(d) 6
12. A metal tank in the shape of a right circular cylinder is one-fourth full of water. If 80 mL of water is added, it will be one-third full. What is the volume of the tank?
- (a) 960 mL
(b) 320 mL
(c) 240 mL
(d) It can't be determined from the information given.
13. Simplify the expression

$$\left(\frac{x^2y^{-3}}{2z^4}\right)^{-5}$$

- (a) $\frac{32z}{x^3y^8}$
(b) $\frac{y^{15}z^{20}}{32x^3}$

- (c) $\frac{32y^{15}z^{20}}{x^{10}}$
(d) $\frac{32y^{15}}{x^{10}z^{20}}$
14. Find the second term in the expansion of $(x + y)^{25}$.
- (a) $2300x^{22}y^3$
(b) $300y^{23}x^2$
(c) $300x^{23}y^2$
(d) $25x^{24}y$
15. If three consecutive integers satisfy the property that twice the product of the first two is ten more than the product of the last two, what is the smallest possible value of the smallest integer?
- (a) -3
(b) -1
(c) 1
(d) 5
16. Machine A, working at a constant rate, produces 36 widgets in a certain number of minutes. Machine B produces three more widgets per minute than machine A. If machine A takes 2 minutes more than machine B to produce 36 widgets, how many widgets does machine B produce per minute?
- (a) 3
(b) 6
(c) 9
(d) 12
17. What two numbers come next in the following sequence:
4, 8, 9, 18, 14, 28, ...
- (a) 19, 38
(b) 20, 40
(c) 23, 46
(d) 25, 50
18. Ten people are attending a meeting. If each person shakes hands with each other person exactly once, how many handshakes will occur?
- (a) 20
(b) 45
(c) 90
(d) 100
19. Brooke, Eric, and Trevor share an office. Trevor is in the office every ninth day, Eric is in the office every fourth day, and Brooke is in the office every sixth day. How often are all three people in the office on the same day?
- (a) Every 12 days
(b) Every 18 days
(c) Every 24 days
(d) Every 36 days

20. A plane took 7 hours to fly 875 miles against the wind, but only 5 hours to return to its starting point. What is the average speed of the wind?
- (a) 150 miles per hour
 (b) 75 miles per hour
 (c) 50 miles per hour
 (d) 25 miles per hour
21. In reduced form, a fraction has the value $\frac{1}{2}$. If 2 is added to the numerator and 1 is subtracted from the denominator, the resulting fraction is $\frac{4}{7}$ in reduced form. What is the original fraction?
- (a) $\frac{4}{8}$
 (b) $\frac{9}{18}$
 (c) $\frac{18}{36}$
 (d) $\frac{20}{40}$
22. A palindrome is a number whose digits read the same forward and backward, like 121. A palindromic square is a palindrome whose square is also a palindrome. For example, 11 is a palindromic square since $11^2 = 121$. Which of the following cannot be the first digit of a palindromic square with more than 1 digit?
- (a) 2
 (b) 3
 (c) 4
 (d) Neither 3 nor 4 can be the first digit of a palindromic square with more than 1 digit.
23. Find the largest possible integer n so that
$$\frac{1005!}{10^n}$$
 is divisible by 10.
- (a) 250
 (b) 249
 (c) 248
 (d) 247
24. Solve for x :
$$x^{(x^3)} = 3.$$
- (a) $3^{\frac{1}{3}}$
 (b) $3^{\frac{1}{2}}$
 (c) $-3^{\frac{1}{2}}$
 (d) There is no solution.
25. Simplify
$$(x + 2i)(x + y + i).$$
- (a) $x^2 + xy + 3ix + 2iy - 2$
 (b) $x^2 + xy + 3ix + 2iy + 2$
 (c) $x^2 + xy + ix + 2iy + 2$
 (d) $x^2 + xy + ix + 2iy - 2$

26. Jay tiled a 15 ft. by 21 ft. room with 1 ft^2 tiles. When he finished, he drew both diagonals connecting opposite sides of the room. What is the total number of tiles that the diagonals passed through?
- (a) 66
 - (b) 65
 - (c) 64
 - (d) 63
27. The land of Xod has coins that are regular triangular pyramids. The four faces are labeled N, G, H, and S. A Xodian is tossing two coins. What is the probability that both coins land with the same side facing down?
- (a) $\frac{1}{2}$
 - (b) $\frac{1}{4}$
 - (c) $\frac{1}{6}$
 - (d) $\frac{1}{8}$
28. If the length of each side of a triangle is increased by 20%, then the area is increased by what percent?
- (a) 40%
 - (b) 44%
 - (c) 48%
 - (d) 52%
29. What is the smallest positive integer n such that if S is a set containing n or more distinct integers, then there must be three integers in S whose sum is divisible by 3?
- (a) 3
 - (b) 4
 - (c) 5
 - (d) 6
30. Suppose the operation \circ is defined by $a \circ b = a - b + ab$. If $3 \circ x = 23$, what is x ?
- (a) 10
 - (b) 12
 - (c) 13
 - (d) none of the above
31. The number 0.125 (base 10) is represented by which of the following base 2 fractions?
- (a) 0.001_2
 - (b) 0.01_2
 - (c) 0.1_2
 - (d) none of the above
32. If a , b , and c are real numbers such that $a - 7b + 8c = 4$ and $8a + 4b - c = 7$, then what is $a^2 - b^2 + c^2$?
- (a) 0
 - (b) 1
 - (c) 4

- (d) 8
33. Lauren, Mary, Nick, and Oscar are good friends. Oscar had no money, but the others did. Mary gave Oscar one-fifth of her money, Lauren gave Oscar one-fourth of her money, and Nick gave Oscar one-third of his money. Each gave Oscar the same amount of money. What fractional part of the group's money does Oscar now have?
- (a) $\frac{1}{10}$
(b) $\frac{1}{4}$
(c) $\frac{1}{3}$
(d) $\frac{2}{5}$
34. Twelve friends met for dinner at Oscar's Overstuffed Oyster House, and each ordered one meal. The portions were so large, there was enough food for 18 people. If they share, how many meals should they have ordered to have just enough for the 12 of them?
- (a) 15
(b) 10
(c) 9
(d) 8
35. Sam usually leaves his cell phone on. If his phone is on but he is not actually using it, the battery will last for 120 hours. If he is using it constantly, the battery will last for only 3 hours. Since the last recharge, his phone has been on for 9 hours and during that time he has used it for 90 minutes. If he doesn't talk any more, but leaves the phone on, how many more hours will the battery last?
- (a) 24
(b) 48
(c) 52.5
(d) 60.5
36. How many three digit numbers satisfy the property that the middle digit is the average of the first and last digits?
- (a) 42
(b) 43
(c) 44
(d) 45
37. Frank buys 13 pencils and 3 erasers for \$1.00. A pencil costs more than an eraser, and both items cost a whole number of cents. What is the total cost, in cents, of one pencil and one eraser?
- (a) 10
(b) 12
(c) 15
(d) 18
38. The ratio of the area of a square inscribed in a semicircle to the area of the square inscribed in the entire circle is
- (a) 1:2
(b) 2:3

- (c) 2:5
(d) 3:4
39. The hypotenuse c and one side b of a right triangle are consecutive integers. The square of the other leg of the triangle is
- (a) bc
(b) $\frac{c}{b}$
(c) $c + b$
(d) $c - b$
40. A line through the point $(-a, 0)$ cuts from the second quadrant a triangular region with area T . The equation of the line is
- (a) $2Tx + a^2y + 2aT = 0$
(b) $2Tx - a^2y + 2aT = 0$
(c) $2Tx + a^2y - 2aT = 0$
(d) $2Tx - a^2y - 2aT = 0$

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PART I ANSWERS

1. d
2. a
3. d
4. a
5. b
6. a
7. c
8. b
9. b
10. c
11. d
12. a
13. c
14. d
15. a
16. c
17. a
18. b

- 19. d
- 20. d
- 21. c
- 22. d
- 23. b
- 24. a
- 25. a
- 26. d
- 27. b
- 28. b
- 29. c
- 30. a
- 31. a
- 32. b
- 33. b
- 34. d
- 35. c
- 36. d
- 37. a
- 38. c
- 39. c
- 40. b

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PART II

1. After a $p\%$ price reduction, what increase does it take to restore the original price?
 - (a) $\frac{p}{1-p}\%$
 - (b) $(100 - p)\%$
 - (c) $\frac{100p}{100+p}\%$
 - (d) $\frac{100p}{100-p}\%$

2. A population starts with a single amoeba. For this one and for the generations thereafter, there is a probability of $\frac{3}{4}$ that an individual amoeba will split to create two amoebas, and a $\frac{1}{4}$ probability that it will die out without producing offspring. What is the probability that the family tree of the original amoeba will go on for ever?
 - (a) $\frac{1}{2}$
 - (b) $\frac{1}{3}$
 - (c) $\frac{2}{3}$
 - (d) $\frac{3}{4}$

3. At a party, every two people shook hands once. How many people attended the party if there were 66 handshakes?
 - (a) 12
 - (b) 22
 - (c) 33
 - (d) 65

4. Given that $f(x) = (x^5 - 1)(x^3 + 1)$ and $g(x) = (x^2 - 1)(x^2 - x + 1)$, and $h(x)$ is a polynomial such that $f(x) = g(x)h(x)$, what is $h(1)$?
 - (a) 0
 - (b) 3
 - (c) 5
 - (d) undefined

5. A circle of radius 6 is inscribed in a regular hexagon. If the area of the hexagon is $x\sqrt{3}$, what is x ?
 - (a) 75
 - (b) 72
 - (c) 71
 - (d) 70

6. The sequence 1, x , 6, y is an arithmetic progression. What is $x + y$?
 - (a) 3.5
 - (b) 5
 - (c) 8.5
 - (d) 12

7. The positive integers 30, 72, and N have the property that the product of any two of them is divisible by the third. What is the smallest possible value of N ?
- (a) 48
 - (b) 60
 - (c) 90
 - (d) 144
8. Sam the goat is tied to the corner of a 4 ft. by 6 ft. shed on a leash that is 6 ft. long. How much area does Sam have in which to play if he can only go around the outside of the shed?
- (a) 28π square feet
 - (b) 36π square feet
 - (c) $\frac{109}{4}\pi$ square feet
 - (d) 16π square feet
9. A rectangle has sides of integer length (when measured in cm) and an area of 36 cm^2 . What is the largest possible perimeter of the rectangle?
- (a) 74 cm
 - (b) 40 cm
 - (c) 30 cm
 - (d) 26 cm
10. If $a < b$, then $3^2 + 4^2 + 5^2 + 12^2 = a^2 + b^2$ is true for only one pair of positive integers (a, b) . Determine $a + b$.
- (a) 24
 - (b) 22
 - (c) 18
 - (d) 8
11. What is the value of $\log_2(\log_2(\log_2(16)))$?
- (a) 4
 - (b) 3
 - (c) 2
 - (d) 1
12. For how many primes p is $p^2 + 3p - 1$ also prime?
- (a) 4
 - (b) 3
 - (c) 1
 - (d) 0
13. At how many points does

$$y = (x^2 - 5x + 9)(x^2 - 8x + 16)(x^2 - 7x + 9)$$

intersect the x -axis?

- (a) 1

- (b) 3
(c) 4
(d) 5
14. Find all real numbers x such that $x = \sqrt{x^2 - 2x + 1} - \sqrt{x + 1}$.
- (a) 0
(b) $\frac{5}{4}$
(c) 1
(d) both (a) and (b) satisfy the equation
15. George's car gets 5 more miles per gallon during highway driving than during city driving. On a recent trip, he drove 120 miles on the highway and 150 miles in the city and used a total of 10 gallons of gasoline. How many miles per gallon does his car get during city driving?
- (a) 8
(b) 20
(c) 25
(d) 30
16. What is the largest value for $|x^2 - 4|$ if $|x + 2| \leq 0.1$?
- (a) 0
(b) 0.39
(c) 0.4
(d) 0.41
17. Evaluate
- $$\frac{2i}{\frac{1}{10-5i} - \frac{1}{10+5i}}.$$
- (a) $12.5i$
(b) 12.5
(c) $25i$
(d) 25
18. A company sells peanut butter in cylindrical jars. Marketing research suggests that using wider jars will increase sales. If the diameter of the jars is increased by 25% without altering the volume, by what percent must the height be decreased?
- (a) 10
(b) 25
(c) 36
(d) 50
19. Jerry had an average score of 85 on his first eight quizzes and an average score of 81 on his first nine quizzes. What was his score on the ninth quiz?
- (a) 49
(b) 50
(c) 51

- (d) 53
20. $f(x)$ and $g(x)$ are linear functions such that for all x , $f(g(x)) = g(f(x)) = x$. If $f(0) = 4$ and $g(5) = 4$, compute $f(2008)$.
- (a) 508
(b) 506
(c) 504
(d) 502
21. How many real solutions does the equation $|2 - |1 - |x|| = 1$ have?
- (a) 1
(b) 2
(c) 3
(d) 5
22. The manager of a store specializing in selling teas decides to experiment with a new blend. She will mix Earl Gray tea that sells for \$5 per pound with some orange pekoe tea that sells for \$3 per pound to obtain 100 pounds of the new blend. The selling price of the new blend is to be \$4.50 per pound and there is to be no change in revenue from selling the new blend versus selling the other types. How many pounds of Earl Gray are required?
- (a) 70
(b) 75
(c) 80
(d) 85
23. A color-blind individual has 16 pairs of socks, 10 identical red pairs and 6 identical navy blue pairs. After washing his socks, he just throws them in the sock drawer without pairing them up. If he randomly selects two socks, what is the probability that they will be the same color?
- (a) $\frac{95}{248}$
(b) $\frac{33}{248}$
(c) $\frac{16}{31}$
(d) $\frac{1}{2}$
24. The product of two positive integers is 25 times their quotient. Which of the following statements is true?
- (a) The sum of the numbers is at least 10.
(b) The difference of the numbers is at most 10.
(c) One of the numbers is 5.
(d) None of the above is true.
25. Bob, Mary, and Sue all have birthdays on the same day. Bob's present age is two years less than the sum of Sue's and Mary's present ages. In five years, Bob will be twice as old as Mary will be then. Two years ago, Mary was one-half as old as Sue was. What is the sum of their ages now?
- (a) 48
(b) 39
(c) 32

- (d) 25
26. The two lines given by $3x + 2y = 8$ and $ax - 8y = 9$ are parallel. What is the value of a ?
- (a) -12
(b) -8
(c) $\frac{16}{3}$
(d) $-\frac{16}{3}$
27. A chocolate bar is composed of twelve pieces. How many snaps will it require to separate all of the pieces?
- (a) 12
(b) 11
(c) 10
(d) 9
28. The sum of a number and 8 times its reciprocal is 6. Find the largest of all such possible numbers.
- (a) 2
(b) 3
(c) 4
(d) 5
29. If the number 86 in base ten is represented as 321 in base b , then the number 123 in base b can be represented in base ten by what number?
- (a) 12
(b) 25
(c) 35
(d) 38
30. How many integers satisfy the inequality $x^2 + 8x < 20$?
- (a) 12
(b) 11
(c) 10
(d) 1
31. At Springfield University, there are 10,000 students, and the number of male and female students is equal. Each student is enrolled in either the Arts program or the Science program (but not in both). If 60% of the students are in the Arts program and 40% of the Science students are male, how many of the arts students are female?
- (a) 1,600
(b) 2,400
(c) 2,600
(d) 3,400
32. A positive integer is called multiplicatively perfect if it is equal to the product of its proper divisors. For example, 10 is multiplicatively perfect, because its proper divisors are 1, 2, and 5, and $1 \times 2 \times 5 = 10$. How many multiplicatively perfect numbers are there between 2 and 30?

- (a) 9
(b) 8
(c) 7
(d) 6
33. Ben and Anna each have some CDs. If Anna gives six of her CDs to Ben, he would then have twice as many CDs as Anna. If, instead, Anna gets six CDs from Ben, then both would have the same number of CDs. What is the total number of CDs that Ben and Anna have?
- (a) 18
(b) 30
(c) 42
(d) 72
34. When 14 is divided by 5, the remainder is 4. When 14 is divided by a positive integer n , the remainder is 2. For how many different values of n is this possible?
- (a) 1
(b) 2
(c) 3
(d) 4
35. If a and b are nonzero real numbers such that $a^2 + b^2 = 8ab$, find the value of
- $$\left| \frac{a+b}{a-b} \right|.$$
- (a) $\frac{\sqrt{15}}{3}$
(b) $\frac{4\sqrt{3}}{3}$
(c) $\sqrt{2}$
(d) none of the above
36. An 8 ounce cup of yogurt costs \$0.72. One way to raise the consumer's cost of the yogurt is to decrease the size of the cup, while keeping the price the same. If the new cup size is 6 ounces, by what percent has the consumer's cost increased?
- (a) 25%
(b) $33\frac{1}{3}\%$
(c) 50%
(d) $66\frac{2}{3}\%$
37. If $x - 1$, $x^2 - 1$, $x^3 - 1$, and $x^4 - 1$ are all factors of a polynomial $p(x)$, what is the smallest possible degree of $p(x)$?
- (a) 5
(b) 6
(c) 7
(d) 10

38. What is the sum of the y -components of the real ordered pair solutions (x, y) of the system of equations

$$\begin{aligned}4^{xy+4} &= 8^{x^2-y-1} \\ y &= x + 1?\end{aligned}$$

- (a) 5
 - (b) 6
 - (c) 7
 - (d) 8
39. In a survey of 69 people, only 9 liked all three of brands A, B, and C; 12 didn't like any of the three; 9 liked only A; 30 disliked A but liked at least one of the other two. If 15 liked exactly two of the three, 12 liked only B, and 31 liked C, how many liked A and B but not C?
- (a) 4
 - (b) 5
 - (c) 7
 - (d) 9
40. You are going on vacation and have asked your neighbor to water your sick plant. The plant is so sick that it will die even after watering with probability 0.2 and without watering with probability 0.8. You are 90% sure that your neighbor will water the plant. What are the chances that your plant will die while you are on vacation?
- (a) 18%
 - (b) 26%
 - (c) 50%
 - (d) 74%

MAD HATTER MARATHON 9-10
PART II ANSWERS

- 1. d
- 2. c
- 3. a
- 4. c
- 5. b
- 6. d
- 7. b
- 8. a
- 9. a
- 10. c
- 11. d
- 12. c

13. b
14. a
15. c
16. d
17. d
18. c
19. a
20. b
21. d
22. b
23. c
24. c
25. a
26. a
27. b
28. c
29. d
30. b
31. c
32. a
33. d
34. d
35. a
36. b
37. b
38. c
39. b
40. b