

# Problem of the Month

## March 2009

Starting this month, we will post a problem every month. You will have until the last date of that month to solve the problem. Solutions can be either

1. written neatly on a sheet of paper and turned in to me or Dr. Oscar Vega at my (PB 347) or his (PB 352) office (simply slide through the door or put it in the white mailbox outside Dr. Vega's office), or
2. typed up using your favorite text editing software (LaTeX preferred) and then turned in to me or him via email at [asabuwala@csufresno.edu](mailto:asabuwala@csufresno.edu) or [ovega@csufresno.edu](mailto:ovega@csufresno.edu).

At the end of the month, we will review all the turned in solutions and post the names of the individuals who have turned in complete correct solutions.

### Problem for March 2009

A sequence of ellipses  $E_1, E_2, \dots, E_n, \dots$  is constructed as follows:

Ellipse  $E_n$  is drawn so as to touch ellipse  $E_{n-1}$  at the extremities of the major axis of  $E_{n-1}$  and to have its foci at the extremities of the minor axis of  $E_{n-1}$ .

Show that if  $e_n$  denotes the eccentricity of the  $n^{\text{th}}$  ellipse, then out of the two sequences  $\{e_1, e_3, e_5, \dots\}$  and  $\{e_2, e_4, e_6, \dots\}$  one is monotonic increasing and the other is monotonic decreasing and that both of them converge to  $1/\tau$ , where  $\tau$  is the golden ratio.