Introduction to logarithmically convex functions and applications

Abstract

A positive function f defined on a convex set I is logarithmically convex or simply log convex if the function

$$g := \log f$$

is convex, i.e.,

$$\log f\{\alpha x + (1-\alpha)y\} \le \alpha \log f(x) + (1-\alpha)\log f(y)$$

for $x, y \in I,$ and $0 \le \alpha \le 1$.

The collection of log convex functions features pretty interesting functions that are widely used in applied mathematics and in statistics. After mentioning some properties of log convex functions, I will show that if f is a positive function defined on $\mathbb{R}_+$ then

$$\frac{[f(y)]^a}{f(ay)} \le \frac{[f(x)]^a}{f(ax)} \le [f(0)]^{a-1}$$

where $0 \le x \le y$ and $a \ge 1$.

I will then derive some well known inequalities involving the Euler gamma function and the Riemann zeta function.