

Introduction to diffraction pattern invariant one dimensional crystals

Let \mathcal{F} be the Fourier transformation defined on the function space of tempered distributions over \mathbb{C} . Since \mathcal{F} is linear then the equation

$$\mathcal{F}f = \lambda f, \quad \lambda \in \mathbb{C} \quad (1)$$

(where f is a nontrivial tempered distribution), is equivalent to

$$\mathcal{F}^n f = \lambda^n f, \quad n \in \mathbb{N}.$$

It is well known that the order of Fourier transformation is 4, thus the characteristic equation is

$$\lambda^4 - 1 = 0$$

i.e., the eigenvalues of the Fourier transformation are

$$\lambda = 1, -1, i, -i.$$

The collection of distributions that satisfy (1) is nonempty since when

$$\begin{aligned} f(x) &= e^{-\pi x^2}, \\ (\mathcal{F}f)(s) &:= \int_{-\infty}^{\infty} f(x)e^{-2\pi i s x} dx \\ &= e^{-\pi s^2} \\ (\mathcal{F}f)(s) &= f(s). \end{aligned}$$

The aim of this talk is to characterize the subcollection of p -periodic distributions that satisfy (1).