From Grammar to Knot Theory via Logic, Categories, and Quantum Groups

Sean Fulop

December 6, 2007

Abstract

In this prospective and general talk, I will lay out a broad set of mathematical interconnections that provide a research program in the proof theory of grammar. I begin with something simple, namely classical propositional logic. A very interesting "matrix method" for checking the provability of a formula in this logic was developed by Wallen (1990); it is a simple graphical criterion deriving from an arrangement of the logical atoms found within the formula. The matrix of a formula can be viewed as a "compressed proof object," since it eliminates so much logical machinery found in ordinary proofs. The main goal of this research program is to develop a general theory of compressed proof objects for a variety of logics.

Why would a linguist pursue such a thing? Because some logics can serve as engines of "grammatical deduction," allowing the proof that a certain structured sequence of words is a sentence in a language. I will show how such logics are close relatives of another species, Linear Logic (Girard 1987), which also has a developed theory of its compressed proof objects which are called "proof nets." Such proof objects for grammatical logics either haven't been discovered yet, or are entirely cumbersome, and this is because a general theory of how to create compressed proof objects has not been formulated. Grammatical proofs are quite unwieldy, and do not admit efficient computation except by some ad hoc tricks. In order to propose such a mechanism as a true cognitive engine behind the production and comprehension of sentences, it is imperative that sentences can be proven with some degree of efficiency.

Now Linear Logic's proof nets are the best understood variety of compressed proof object; it is known, for instance, that they are a species of knot, and so their mathematical properties (and thus the properties of Linear Logic) are connected to knot theory. The theory of "quantum groups" (quasitriangular Hopf algebras) is also connected to knot theory, and it is known how Hopf algebras can serve as models (using category theory) of Linear Logics and their proof nets. This bunch of results is not yet applicable to the grammatical logics, but to me everything desired seems close at hand. I will try to outline some of the mathematical steps still to be taken in order to complete this highly interesting and interconnected theory.