The Mystery of the Euler-Cauchy Surprise Unraveled

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Abstract

The Euler-Cauchy differential equation is one of the first and simplest forms of a non-constant coefficient ODE that is encountered in an undergraduate differential equations class. For a non-homogeneous Euler-Cauchy equation, the particular solution takes one of several possible forms which are usually determined by using the method of undetermined coefficients or variation of parameters. Last week, I got a chance to hear about the "Euler-Cauchy surprise" from Dr. Doreen De Leon where she presented a few examples of the surprising discovery that she made for the case of a polynomial function in t as the non-homogeneity for second-order Euler-Cauchy equations. In this talk, I unravel the mystery behind this surprising form of the particular solution for the most general case, an n^{th} order Euler-Cauchy equation with a polynomial right hand side, by presenting a complete proof of the form of the particular solution. In addition, I will also present a formula that can be used to compute the unknown coefficient in the form of the particular solution.